Properties, Constants & Vector Equations

Properties of Water

$$\rho = 10^3\,\text{kg/m}^3$$
$$\mu = 10^{-3}\,\text{Pa s}$$
$$\gamma = 72.9\,\text{mN/m (air interface)}$$

Properties of Air at 27°C and 100 kPa

$$\rho = 1.18\,\text{kg/m}^3$$
$$\mu = 1.86\times 10^{-5}\,\text{Pa s}$$
$$c = 347\,\text{m/s}$$
$$\hat{C}_p/\hat{C}_v = 1.40$$

Unit Conversions

$$1\,\text{m} = 3.2808\,\text{ft}$$
$$1\,\text{kPa} = 0.1450\,\text{psi}$$
$$1\,\text{atm} = 101.3\,\text{kPa} = 14.7\,\text{psi}$$
$$1\,\text{kg} = 2.2056\,\text{lbm}$$
$$1\,\text{slug} = 32.174\,\text{lbm}$$
$$1\,\text{lbf} = 4.448\,\text{N} = 1\,\text{slug}\cdot\text{ft/s}^2$$

Dimensional Constants

$$g = 9.807\,\text{m/s}^2 = 32.17\,\text{ft/s}^2$$
$$R = 8314\,\text{Pa m}^3/(\text{kg} - \text{mol}\,K)$$

Minor Losses

Object	K_L
Sudden Contraction	0.44
Sudden Expansion	1.0
Orifice	2.6
90° bend	0.75
Gate valve (open)	0.17
Gate valve (half open)	4.5

Orifice coefficient

Type	C_o
Orifice meter	0.61
Flow nozzle	0.96
Venturi meter	0.98

Drag Coefficient

Object	Re < 1	Re > 2×10^5
Sphere	24/Re	0.2
Disk	$64/(\pi\text{Re})$	1.17
Cylinder	$\dfrac{8\pi}{\text{Re}\ln(7.4/\text{Re})}$	0.3

Expanded Vector Equations – Cartesian Coordinates

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

$$\rho\left[\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right] = -\frac{\partial \mathcal{P}}{\partial x} + \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right]$$
$$\rho\left[\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right] = -\frac{\partial \mathcal{P}}{\partial y} + \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right]$$
$$\rho\left[\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right] = -\frac{\partial \mathcal{P}}{\partial z} + \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right]$$

$$\Gamma_{xx} = \frac{\partial v_x}{\partial x}, \quad \Gamma_{yy} = \frac{\partial v_y}{\partial y}, \quad \Gamma_{zz} = \frac{\partial v_z}{\partial z}$$
$$\Gamma_{xy} = \Gamma_{yx} = \frac{1}{2}\left[\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right]$$
$$\Gamma_{yz} = \Gamma_{zy} = \frac{1}{2}\left[\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right]$$
$$\Gamma_{zx} = \Gamma_{xz} = \frac{1}{2}\left[\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right]$$

$$\tau_{xx} = 2\mu\frac{\partial v_x}{\partial x}, \quad \tau_{yy} = 2\mu\frac{\partial v_y}{\partial y}, \quad \tau_{zz} = 2\mu\frac{\partial v_z}{\partial z}$$
$$\tau_{xy} = \tau_{yx} = \mu\left[\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right]$$
$$\tau_{yz} = \tau_{zy} = \mu\left[\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right]$$
$$\tau_{zx} = \tau_{xz} = \mu\left[\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right]$$

Expanded Vector Equations – Cylindrical Coordinates

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

$$\rho\left[\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right] = -\frac{\partial \mathcal{P}}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right]$$

$$\rho\left[\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right] = -\frac{1}{r}\frac{\partial \mathcal{P}}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right]$$

$$\rho\left[\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right] = -\frac{\partial \mathcal{P}}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right]$$

$$\Gamma_{rr} = \frac{\partial v_r}{\partial r},\ \Gamma_{\theta\theta} = \left[\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right],\ \Gamma_{zz} = \frac{\partial v_z}{\partial z}$$

$$\Gamma_{r\theta} = \Gamma_{\theta r} = \frac{1}{2}\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]$$

$$\Gamma_{\theta z} = \Gamma_{z\theta} = \frac{1}{2}\left[\frac{\partial v_\theta}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \theta}\right]$$

$$\Gamma_{zr} = \Gamma_{rz} = \frac{1}{2}\left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right]$$

$$\tau_{rr} = 2\mu\frac{\partial v_r}{\partial r},\ \tau_{\theta\theta} = 2\mu\left[\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right],\ \tau_{zz} = 2\mu\frac{\partial v_z}{\partial z}$$

$$\tau_{r\theta} = \tau_{\theta r} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]$$

$$\tau_{\theta z} = \tau_{z\theta} = \mu\left[\frac{\partial v_\theta}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \theta}\right]$$

$$\tau_{zr} = \tau_{rz} = \mu\left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right]$$

Expanded Vector Equations – Spherical Coordinates

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\rho v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\rho v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\rho v_\phi) = 0$$

$$\rho\left[\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right] =$$

$$-\frac{\partial \mathcal{P}}{\partial r} + \mu\left[\nabla^2 v_r - \frac{2}{r^2}v_r - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} - \frac{2}{r^2}v_\theta\cot\theta - \frac{2}{r^2\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right]$$

$$\rho\left[\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta}{r} - \frac{v_\phi^2\cot\theta}{r}\right] =$$

$$-\frac{1}{r}\frac{\partial \mathcal{P}}{\partial \theta} + \mu\left[\nabla^2 v_\theta + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r^2\sin^2\theta} - \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial v_\phi}{\partial \phi}\right]$$

$$\rho\left[\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r}{r} + \frac{v_\theta v_\phi\cot\theta}{r}\right] =$$

$$-\frac{1}{r\sin\theta}\frac{\partial \mathcal{P}}{\partial \phi} + \mu\left[\nabla^2 v_\phi - \frac{v_\phi}{r^2\sin^2\theta} + \frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial \phi} + \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial v_\theta}{\partial \phi}\right]$$

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\left(\frac{\partial^2}{\partial \phi^2}\right)$$

$$\Gamma_{rr} = \frac{\partial v_r}{\partial r}$$
$$\Gamma_{\theta\theta} = \left[\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right]$$
$$\Gamma_{\phi\phi} = \left[\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta \cot\theta}{r}\right]$$
$$\Gamma_{r\theta} = \Gamma_{\theta r} = \frac{1}{2}\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]$$
$$\Gamma_{\theta\phi} = \Gamma_{\phi\theta} = \frac{1}{2}\left[\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right]$$
$$\Gamma_{\phi r} = \Gamma_{r\phi} = \frac{1}{2}\left[\frac{1}{r\sin\theta}\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right]$$

$$\tau_{rr} = 2\mu\frac{\partial v_r}{\partial r}$$
$$\tau_{\theta\theta} = 2\mu\left[\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right]$$
$$\tau_{\phi\phi} = 2\mu\left[\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta \cot\theta}{r}\right]$$
$$\tau_{r\theta} = \tau_{\theta r} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]$$
$$\tau_{\theta\phi} = \tau_{\phi\theta} = \mu\left[\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right]$$
$$\tau_{\phi r} = \tau_{r\phi} = \mu\left[\frac{1}{r\sin\theta}\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right]$$

I. Phenomenology & Dimensional Analysis

$$x_{n+1} = x_n - f(x_n)/f'(x_n)$$

$$\tau = \mu s$$

$$\tau_w = -\frac{A}{C}\frac{\Delta\mathcal{P}}{L}$$

$$D_H = \frac{4A}{C}$$

$$f = \frac{-\Delta\mathcal{P}}{L}\frac{D_H}{2\rho U^2}$$

$$f = \frac{16}{\text{Re}}$$

$$\frac{1}{\sqrt{f}} = -4\log\left(\frac{1.26}{\text{Re}\sqrt{f}} + \frac{k}{3.7D}\right)$$

$$f = \left\{3.6\log\left[\frac{6.9}{\text{Re}} + \left(\frac{k}{3.7D}\right)^{1.11}\right]\right\}^{-2}$$

$$\tau = ms^n$$

$$\text{Re}_{\text{PL}} = \frac{8\rho U^{2-n}D^n}{m\left[\frac{2(3n+1)}{n}\right]^n}$$

$$f = \frac{16}{\text{Re}_{\text{PL}}}$$

$$\frac{1}{\sqrt{f}} = \frac{4.0}{n^{0.75}}\log\left(\text{Re}_{\text{PL}}\, f^{1-n/2}\right) - \frac{0.4}{n^{1.2}}$$

$$C_D = \frac{2F_D}{\rho U^2 A_\perp}$$

$$C_f = \begin{cases} 1.328\,\text{Re}^{-1/2}, & 10^2 < \text{Re} < 3\times 10^5 \\ \dfrac{0.455}{(\log \text{Re})^{2.58}} - \dfrac{1050}{\text{Re}}, & \text{Re} > 3\times 10^5 \end{cases}$$

$$F_B = \frac{\pi}{6}(\rho - \rho_o)gD^3$$

$$\text{Re}^2 C_D = \frac{4}{3}\frac{\Delta\rho}{\rho}\frac{gD^3}{\nu^2}$$

II. Fundamentals & Differential Theory

$$\nabla P = \rho \boldsymbol{g}$$

$$\boldsymbol{F}_P = -\int \boldsymbol{n} P \, dS$$

$$\boldsymbol{w} = \nabla \times \boldsymbol{v}$$

$$v_x = \frac{\partial \psi}{\partial y} \qquad v_y = -\frac{\partial \psi}{\partial x}$$

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \qquad v_\theta = -\frac{\partial \psi}{\partial r}$$

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \nabla f$$

$$\boldsymbol{a} = \frac{D\boldsymbol{v}}{Dt}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0$$

$$\nabla \cdot \boldsymbol{v} = 0$$

$$\frac{1}{L}\frac{dL}{dt} = \boldsymbol{p} \cdot \boldsymbol{\Gamma} \cdot \boldsymbol{p}$$

$$\boldsymbol{\Gamma} = \frac{1}{2}\left[\nabla \boldsymbol{v} + (\nabla \boldsymbol{v})^T\right]$$

$$\boldsymbol{F} = A\boldsymbol{n} \cdot \boldsymbol{\sigma}$$

$$\boldsymbol{\sigma} = -\boldsymbol{\delta} P + \boldsymbol{\tau}$$

$$\boldsymbol{F} = \int_S \boldsymbol{n} \cdot \boldsymbol{\sigma} dS$$

$$\boldsymbol{F}_D = -\int_S \boldsymbol{e}_z \cdot \boldsymbol{n} P \, dS + \int_S \boldsymbol{n} \cdot \boldsymbol{\tau} \cdot \boldsymbol{e}_z \, dS$$

$$\boldsymbol{\tau} = 2\mu\Gamma$$

$$\boldsymbol{\tau} = \mu\left[\nabla \boldsymbol{v} + (\nabla \boldsymbol{v})^T\right]$$

$$f = \frac{2\tau_w}{\rho U^2}$$

$$C_D = \frac{2F_D/A_\perp}{\rho U^2}$$

$$U = \frac{1}{A}\int \boldsymbol{v} \, dA$$

$$\tau_{rz} = m\left(\frac{\partial v_z}{\partial r}^n\right)$$

$$\rho\frac{D\boldsymbol{v}}{Dt} = \rho\boldsymbol{g} + \nabla \cdot \boldsymbol{\sigma}$$

$$\rho\frac{D\boldsymbol{v}}{Dt} = \rho\boldsymbol{g} - \nabla P + \mu\nabla^2\boldsymbol{v}$$

$$\nabla\mathcal{P} = \nabla P - \rho\boldsymbol{g}$$

$$0 = -\nabla\mathcal{P} + \mu\nabla^2\boldsymbol{v}$$

$$\rho\frac{D\boldsymbol{v}}{Dt} = -\nabla\mathcal{P}$$

$$\boldsymbol{v} = \nabla\phi$$

$$\nabla^2\phi = 0$$

$$v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} = -\frac{1}{\rho}\frac{\partial \mathcal{P}}{\partial x} + \nu\frac{\partial^2 v_x}{\partial y^2}$$

$$v_x(x,y) = U\left[\frac{3}{2}\left(\frac{y}{\delta(x)}\right) - \frac{1}{2}\left(\frac{y}{\delta(x)}\right)^3\right]$$

$$\delta(x) = \sqrt{\frac{840}{39}\frac{\nu x}{U}}$$

$$\rho\frac{D\langle\boldsymbol{v}\rangle}{Dt} = -\nabla\langle P\rangle + \mu\nabla^2\langle\boldsymbol{v}\rangle + \nabla \cdot \boldsymbol{\tau}^*$$

$$\boldsymbol{\tau}^* = -\rho\langle\boldsymbol{u}\boldsymbol{u}\rangle$$

$$\tau^*_{xy} = \rho\epsilon\frac{\partial\langle v_x\rangle}{\partial y}$$

$$\nabla^2 P = -\nabla \cdot (\boldsymbol{v} \cdot \nabla\boldsymbol{v})$$

$$\frac{df}{dx} = \frac{f_{i+1} - f_i}{\Delta x}$$

$$\frac{d^2 f}{dx^2} = \frac{f_{i+1} - 2f_i + f_{i-1}}{(\Delta x)^2}$$

III. Macroscopic Engineering Systems

$$\frac{d}{dt}\int_{V(t)} \rho \, dV = -\int_{S(t)} \rho(\boldsymbol{v}-\boldsymbol{u})\cdot\boldsymbol{n}\, dS$$

$$\frac{d}{dt}\int_{V(t)} \rho\boldsymbol{v} \, dV = -\int_{S(t)} \rho\boldsymbol{v}(\boldsymbol{v}-\boldsymbol{u})\cdot\boldsymbol{n}\, dS + m\boldsymbol{g} - \int_{S(t)} \boldsymbol{n}P\, dS + \int_{S(t)} \boldsymbol{n}\cdot\boldsymbol{\tau}\, dS$$

$$\frac{dm}{dt} = \sum_i^{\text{inlets}} w_i - \sum_i^{\text{outlets}} w_i$$

$$\frac{d(m\boldsymbol{v})}{dt} = \sum_i^{\text{inlets}} a_i w_i \boldsymbol{v}_i - \sum_i^{\text{outlets}} a_i w_i \boldsymbol{v}_i + \sum_i \boldsymbol{F}_i$$

$$\frac{d}{dt}\int_{V(t)} \rho\left[\frac{v^2}{2}+gh\right] dV + \int_{S(t)} \rho\left[\frac{v^2}{2}+gh\right](\boldsymbol{v}-\boldsymbol{u})\cdot\boldsymbol{n}\, dS = -\int_{S(t)} P(\boldsymbol{n}\cdot\boldsymbol{v})\, dS + \int_{S(t)} \boldsymbol{n}\cdot\boldsymbol{\tau}\cdot\boldsymbol{v}\, dS + \int_{S(t)} P(\nabla\cdot\boldsymbol{v})\, dS - \int_{S(t)} \boldsymbol{\tau}:\nabla\boldsymbol{v}\, dS.$$

$$\left(\frac{bv^2}{2}+\frac{P}{\rho}+gh\right)_{\text{outlet}} - \left(\frac{bv^2}{2}+\frac{P}{\rho}+gh\right)_{\text{inlet}} = \frac{1}{w}\left(W_m - E_v - E_c\right)$$

$$a_i = \begin{cases} 4/3 \text{ for laminar flow} \\ 50/49 \approx 1 \text{ for turbulent flow} \\ 1 \text{ for plug/uniform flow} \end{cases}$$

$$b = \begin{cases} 2 \text{ for laminar flow} \\ 1.06 \approx 1 \text{ for turbulent flow} \\ 1 \text{ for plug/uniform flow} \end{cases}$$

$$E_v = Q\,|\Delta\mathcal{P}|$$

$$h_p = A - BQ^n$$

$$E_{v,tot} = \sum_i K_{L,i} w \frac{U^2}{2} + \frac{2LfwU^2}{D}$$

$$h_p = \Delta H + h_L$$

$$\text{NPSH} = \left(\frac{P}{\rho g}+\frac{v^2}{2g}\right)_{\text{in}} - \frac{P_v}{\rho g}$$

$$K_o = \frac{1}{C_o^2}(1-\beta^2)(1-\beta^4)$$

$$\beta = D_o/D_1$$

$$\int_{P_1}^{P_2} \frac{dP}{\rho} + \Delta\left(\frac{v^2}{2}+gh\right) = \frac{1}{\dot m}\left(W_m - E_v\right)$$

$$\eta_p = \frac{\text{whp}}{\text{bhp}} = \frac{\rho Q g h_p}{Tw}$$

$$\rho = \frac{MP}{RT}$$

$$\eta_t = \frac{\text{bhp}}{\text{whp}} = \frac{Tw}{\rho Q g h_t}$$

$$c^2 = \frac{\gamma RT}{M}, \quad \gamma = \frac{\hat C_p}{\hat C_v}$$

$$W_m = \eta_p T w$$

$$h_p = h_{\max} - \frac{h_{\max}}{Q_{\max}} Q$$

$$\left(\frac{P_0}{P}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_0}{T} = 1 + \left(\frac{\gamma-1}{2}\right)\text{Ma}^2$$