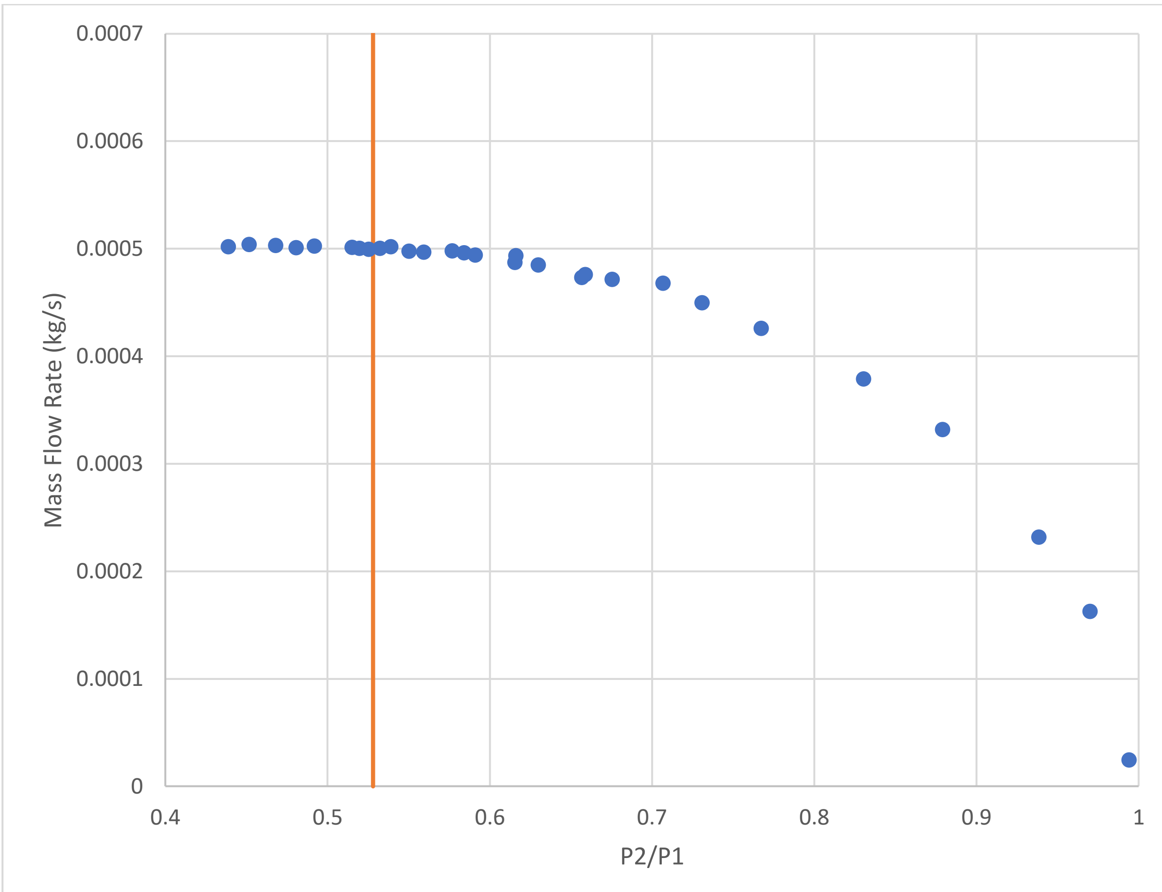
Key

a) The flow in this system is adiabatic so we can use the following equation:

$$1 + \left(\frac{\gamma - 1}{2}\right) \mathrm{Ma}^2 = \frac{T_0}{T} = \left(\frac{P_0}{P}\right)^{\frac{\gamma-1}{\gamma}}$$

Where P_0 is the upstream pressure and P is the downstream pressure. By setting the Mach number equal to 1 (since we want the flow to be choked) and using a heat capacity ratio of 1.4, the pressure ratio (P/P_0) is found to be 0.528.



b) The orange line represents the pressure ratio solved for in part a). Below this pressure ratio, the mass flow rate in constant. This means that, at choked flow, we have the maximum possible mass flow rate that can occur through this orifice. Having choked flow could be useful in reactors that need a guaranteed high amount of cooling water.

$C_d := 0.99 \qquad \gamma := 1.4 \qquad P := 50\ psi + 86\ kPa \qquad T := 25\ °C \qquad MW := 29\ \frac{gm}{mol}$

$\rho := \frac{P}{R \cdot T} \qquad D := 1\ mm \qquad A := \frac{\pi \cdot D^2}{4} \qquad \mu := 18.6 \cdot 10^{-6}\ Pa \cdot s$

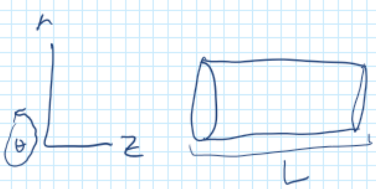
$$m_{dot} := C_d \cdot A \cdot \sqrt{\gamma \cdot \rho \cdot P \cdot MW \cdot \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} = \left(7.844 \cdot 10^{-4}\right) \frac{kg}{s}$$

$$Re := \frac{\rho \cdot \frac{m_{dot}}{\rho \cdot A} \cdot D}{\mu} = 5.369 \cdot 10^{4}$$

Since Re>2100, this flow is turbulent!

c)

III Cylindrical Poiseuille Flow (Pipe Flow)



assumptions

axisymmetric: doesn't depend on θ

fully developed $v_z \neq v_z(z)$ $\quad v_r \neq v_r(\theta) \neq v_z(\theta)$

unidirectional $v_\theta = 0$

(we can leave gravity in since we use $\mathcal{P}$)

steady flow $\frac{\partial}{\partial t} \to 0$

$v_r = 0$

$v_\theta = 0$

$v_z = v_z(r)$

$$\rho\left(\underbrace{\cancel{\frac{\partial v_z}{\partial t}}}_{\text{steady}} + \cancel{v_r}\frac{\partial v_z}{\partial r} + \frac{\cancel{v_\theta}}{r}\frac{\partial v_z}{\partial \theta} + v_z \overbrace{\cancel{\frac{\partial v_z}{\partial z}}}^{\text{fully developed}}\right) = -\frac{\partial \mathcal{P}}{\partial z}$$

$$+ \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\cancel{\frac{\partial^2 v_z}{\partial \theta^2}} + \cancel{\frac{\partial^2 v_z}{\partial z^2}}\right]$$

v_z is function of just r

$$0 = \frac{-\partial p}{\partial z} + \mu \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial v_z}{\partial r}\right)$$

Boundary conditions

$v_z(R) = 0$ from no slip assumption

$|v_z(0)| < \infty$ Symmetry condition

d) Yes, we can solve for the velocity profile with the given conditions and simplifications.