Lec 11 - Supplement : Dimensionless solution of heated wire

solution: $T = T_\infty + \frac{H_v R^2}{4k}\left[1 - \left(\frac{r}{R}\right)^2\right] + \frac{H_v R}{2h}$

$\frac{T - T_\infty}{T_s - T_c} = \theta = \frac{-k}{H_v R^2}(T - T_\infty) \qquad \eta = r/R$

Substitute : $-\theta \cdot \frac{H_v R^2}{k} = \frac{H_v R^2}{4k}\left[1 - \eta^2\right] + \frac{H_v R}{2h}$

$$\theta = -\frac{1}{4}(1 - \eta^2) - \frac{\cancel{H_v R}}{2h} \cdot \frac{k}{\cancel{H_v R^2}}$$

$$\theta = -\frac{1}{4}(1 - \eta^2) - \frac{k}{2hR} \qquad \frac{1}{Bi} = \frac{k}{hR}$$

$$= -\frac{1}{4}(1 - \eta^2) - \frac{1}{2Bi}$$

$$\boxed{\theta = \frac{1}{4}(\eta^2 - 1) - \frac{1}{2Bi}}$$

* This is valid, but is inconvenient to plot. Define a different dimensionless temperature:

$$\tilde{T} = \frac{T - T_\infty}{T_c - T_\infty} = \frac{T - T_\infty}{\frac{H_v R^2}{4k} + \frac{H_v R}{2h}} = \frac{T - T_\infty}{\frac{H_v R^2}{4k}\left(1 + \frac{2k}{hR}\right)} = \frac{T - T_\infty}{\frac{H_v R^2}{4k}\left(1 + \frac{2}{Bi}\right)}$$

whole temp domain from inside to external

$$\tilde{T}\left[\frac{H_v R^2}{4k}\left(1 + \frac{2}{Bi}\right)\right] = \frac{H_v R^2}{4k}\left[1 - \eta^2\right] + \frac{H_v R}{2h}$$

$$\tilde{T}\left(1 + \frac{2}{Bi}\right) = (1 - \eta^2) + \underbrace{\frac{H_v R}{2h} \cdot \frac{4k}{H_v R^2}}_{2/Bi}$$

$$\tilde{T} = \frac{(1-\eta^2) + 2/Bi}{1 + 2/Bi} \cdot \frac{Bi}{Bi}$$

$$\boxed{\tilde{T} = \frac{Bi(1-\eta^2) + 2}{Bi + 2}}$$

* As resistors in series:

$$\tilde{T} = \frac{Bi(1-\eta^2)}{Bi+2} + \frac{2}{Bi+2}$$

* Limits:

(1) $Bi \ll 1$ (slow convection, fast conductivity)
$\tilde{T} \to 1$ (uniform temp distribution)

(2) $Bi \gg 1$ (fast convection, slow conduction)
$\tilde{T} \to 1-\eta^2$ (non-uniform temp distribution)