Special Problem 3-2

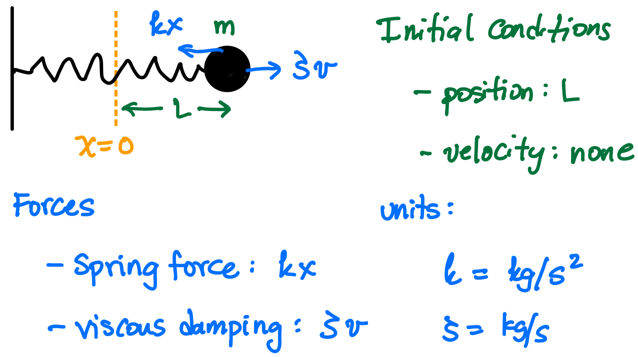
A mass attached to a spring submerged in a viscous liquid that is purturbed from equilibrium is called a "damped harmonic oscillator." The differential equation describing the dynamics of this system is given by

$$m\frac{d^2x}{dt^2} + \zeta\frac{dx}{dt} + kx = 0$$

where x is the position of the mass, t is time, m is the mass, ζ is the viscous drag coefficient, and k is the spring constant. The initial conditions for the system are given by

$$x(t = 0) = L$$
$$\left.\frac{dx}{dt}\right|_{t=0} = 0$$

where L is the length of the initial perturbation from equilibrium.



a). Identify the dependent and independent variables in the system. Determine the units of both variables. Finally, determine a proper scale for both variables. One way to determine a useful scale is to use the equation for a simple harmonic oscillator, $m\ddot{x} + kx = 0$, that omits the viscous damping term.

b). Use the Buckingham Pi theorem to determine the number of dimensionless groups.

c). Using what you know from the previous parts, non-dimensionalize the ODE and initial conditions and determine valid dimensionless groups.

d). Use the scaling analysis that you completed above to describe some insights into dynamics of this system. What is the physical meaning for the scales for the dependent and independent variable? What ratios are expressed in the dimensionless group(s) that you found? What qualitative solutions do you expect for different values of the dimensionless group(s)?

e). Plot the solution to the two ODEs (you may solve these numerically or by hand). Comment briefly on the comparison between the solutions of these ODEs and your scaling analysis of the damped harmonic oscillator.

$$\ddot{x} + 10\dot{x} + x = 0, \quad x(0) = 1, \quad \dot{x}(0) = 0 \tag{1}$$
$$\ddot{x} + 0.4\dot{x} + x = 0, \quad x(0) = 1, \quad \dot{x}(0) = 0 \tag{2}$$