Special Problem A-2

(a) Use Cartesian tensor or index notation to prove that

$$(\boldsymbol{u} \times \boldsymbol{v}) \cdot \boldsymbol{w} = \boldsymbol{u} \cdot (\boldsymbol{v} \times \boldsymbol{w})$$

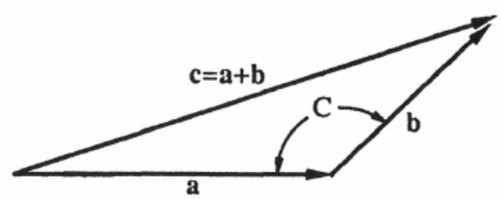
assuming that $\boldsymbol{u}$, $\boldsymbol{v}$, $\boldsymbol{w}$, are vectors in $\mathbb{R}^3$.

(b) Use Cartesian tensor or index notation to prove that

$$\boldsymbol{a} \cdot \boldsymbol{b}\boldsymbol{c} \cdot \boldsymbol{d} = \boldsymbol{b} \cdot \boldsymbol{a}\boldsymbol{d} \cdot \boldsymbol{c}$$

assuming that $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, and $\boldsymbol{d}$ are vectors in $\mathbb{R}^3$.

(c) Use vector properties or Cartesian tensory/index notation as needed to derive the law of cosines. *Hint: Use the figure below and the fact that* $c^2 = \boldsymbol{c} \cdot \boldsymbol{c}$.



$$c^2 = a^2 + b^2 - 2ab \cos C$$

Figure 1.6. Law of cosines for plane triangles.

Credit: D. A. Danielson, "Vectors and Tensors in Engineering and Physics."