Transport Homework Hints

February 13, 2024

1 Question SPA-1

Dot products always drop 2 dimensions from the sum of the dimensions of the multiplicands that go into it. Double dot products always drop 4 dimensions. Additionally, there are a few ways you can write out vectors and tensors. If one way doesn't make sense to you try another method. A tutorial you can use to refresh on this stuff is `https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:matrices/x9e81a4f98389efdf:multiplying-matrices-by-matrices/a/multiplying-matrices`

2 Question SPA-2

If index notation doesn't make sense to you try writing things out in full component notation. As a refresher component notation looks like this:

$$a \cdot b = \sum_i \sum_j a_i b_j (\boldsymbol{e_i} \cdot \boldsymbol{e_j}) = \sum_i \sum_j a_i \delta_{ij} b_j$$

and index notation looks like this:

$$a \cdot b = a_i \delta_{ij} b_j$$

The trick to the first 2 proofs is to use the δ_{ij} which can be used to change the index such that

$$a_i \delta_{ij} b_j = a_i b_i$$

This can be used in both directions so you can change something to the same index and then change it back to two different indexes by reintroducing this δ_{ij}. Pages 613-617 contain all of the information you should need.

3 Question SP1-1

It is as simple as it seems. Just remember that the numbers have been scaled in the table so don't forget to account for those scaling factors when doing the calculations.

4 Question 1-1

$$1 = x_A + x_B, 1 = \omega_A + \omega_B$$

for a binary solution. The gradient of 1 is 0.

5 1-3

This question does not seem to appear in the international edition so watch out for that.

a). The easiest place to start are these equations:

$$\boldsymbol{N}_A = C_A \boldsymbol{v} + \boldsymbol{J}_A = C_A \boldsymbol{v}^{(M)} + \boldsymbol{J}_A^{(M)}$$

$$\boldsymbol{N}_B = C_B \boldsymbol{v} + \boldsymbol{J}_B = C_B \boldsymbol{v}^{(M)} + \boldsymbol{J}_B^{(M)}$$

You should be able to find an expression for $v^{(M)} - v$ from manipulating these equations (think adding or subtracting as well as just rearranging) and then plug the expression back in to get the answer.

b). The key part of this problem is finding a relationship between $\nabla \omega$ and ∇x. Once you have this the rest of the problem is pretty simple.

6 Question 1-4

There are two ways to solve this problem based on two relationships in the book A.8-2 and A.8-7. A.8-7 is easier. You also need to find an expression for F(x,y). For this just solve the original equation for z.

7 Question 2-1

b) You need to use the definition of dU (internal energy) for a closed system. Remember that $V = 1/\rho$.

c) Temperature **cannot** be assumed constant over space. You will need to use the quotient rule for this problem. σ is the B_v term from the generic equation.

d) This might be the trickiest question that you will face in this class or at least the question that the most people get wrong. You need to prove that thermal conductivity (k) is positive. You don't need to solve for k to do this but you will need to get a relationship that you can point to and prove that it can't be negative.

Assume this process is spontaneous and that the control volume is fixed. What does this tell you about entropy?

Start with the equation you have been building in parts a-c and take the volume integral of the whole thing. You will have two terms with a $\boldsymbol{q}$ in them. You can apply the divergence theorem to one of them. Remember this is a closed system.

Also $\nabla T \cdot \nabla T = (\nabla T)^2$

8 Question 2-2

Start with Eq 2.2-1, plug stuff in correctly and simplify. Part a) is really only 1.5 steps until you have an answer. Think about what the prompt is asking. Part b starts from the answer you get in part a and is only 2 steps before you are done. You should end up with a familiar electrochemistry equation.

9 Question 2-4

b) the normal should be pointing out of the bubble, into the liquid. How is velocity defined (think units and derivatives)?

c) ρ_G can be assumed to be small compared to ρ_L

10 Question 3-2

Heterogenous means that the reaction is happening at the wall not in the bulk. The generation term in the conservation equation will therefore be 0 because the reaction is not happening in your control volume.

Pay attention to the whole question and the whole example in the book, there are 3 deliverables for this question: Dimensionless variables and parameters (there are 6 of them), Dimensionless concentration equations, and a dimensionless reaction rate.

Reaction rate = -Boundary flux. Solving your differential equations will give two easy to apply boundary conditions and 2 difficult ones. The two hard constants can only found in a coupled manner. In other words: You need to relate a_A to a_B using your boundary conditions and then plug in your solved differential equation into one of the boundary conditions to solve for each of those constants.

Interfacial balances won't be helpful on this question.

11 Question 3-10

Figure P3-10 is at the bottom of the page where the question is found. Set your zero for x to be the interface between the wall and the heater.

You will have two differential equations that are coupled by a boundary condition that they share; one for the wall and one for the heater. Your heater will have 3 boundary conditions instead of just two, to account for the extra condition we are applying.

12 Question 3-18

The reaction in this problem is not a boundary condition. Remember you are going to be in spherical coordinates. It might be easier to calculate the flux after non-dimensionalizeing things. a) this is asking for a ratio of the system with a reaction to the system without a reaction. One way to remove the reaction from the system is to set the rate constant to 0.

b) You need an expression for flux without convection as well as one for flux with convection. Then you need to divide your convection term by your non-convection flux, which will give a relationship between C_0 and C_b. I don't like this hint but I can't come up with anything better that isn't a walkthrough

c) You need to do Order of magnitude for slow and fast reactions separately and find which one is more likely to meet the condition for a pseudosteady system.

13 Question 3-21ab

a) You might not believe me but part a is a plug and chug so long as you know what to plug. Just notice that part a itself asks two things. You will also need to do a bit of order of magnitude estimation.

b) You need to do a macroscopic balance for each species. Again this problem has a numerical answer involved.

14 Question 3-6

Part a) This question requires you to find four things: x_R, $C_A(x)$, $C_B(x)$, and $C_C(x)$ $C_A(x)$ and $C_B(x)$ are straightforward differential equations with simple boundary conditions.

x_R requires you to do an interfacial balance for A and B. You also know that $R_A = R_B = -R_C$. You can find x_R from the relationship between the fluxes using the derivatives of your two earlier differential equations.

C_C is also simple, but requires two differential equation(similar to 4-1) (remember the reaction only occurs at the boundary). This will have 4 boundary conditions. One of them is that $C_{C,+}(x_R) = C_{C,-}(x_R)$ and another is another interfacial balance but for C this time.

part b) Do an OoM estimate on your differential equation with the given conditions. You will have 2 unknowns C* and δ but only one equation. The second equation comes from an overall conservation equation for A or B.

$$N_A = \int_0^L R_A dx$$

which you can do an OoM estimate on as well.

15 Question 4-8

This question asks you to relate C_A and C_B. To do this write the differential equation

$$\frac{d^2(C_A - C_B)}{dx^2} = 0$$

and solve it with similarly combined boundary conditions. Then plug that in for B in your true differential equation. Then all you have to do is non-dimensionalize it. For part b) Just do perturbation like you learned in class. The final result will be a bunch of terms and polynomial stuff. Don't expect it to look too pretty.

16 Question 4-9

No special tricks. Just non-dimensionalize and do perturbation.

17 Question 5-1

Make sure to transform every term in your differential equation including the 1. There is only one real trick and that is that:

$$\frac{\sqrt{2}}{\lambda_n}\sin\lambda_n x\Big|_{x=0}^{x=1} = \sqrt{2}\frac{(-1)^n}{\lambda_n}$$

Also make sure to keep careful track of your negatives so as not to lose unnecessary points. Your final differential equation will have a homogenous solution with a sinh and a cosh in it.

18 Question 5-12

Should be just a straightforward FT. Your first integration by parts will not go to zero. You will end up with a familiar equation plus an extra term.

Once you solve the differential equation, your solution will be able to be split into two parts, a steady state part and a transient part. This will be helpful when you calculate the flux for part b.

19 Question 5-16a

There is a similar trick to question 5-1. In this case:

$$\sqrt{2}\left(n\pi\frac{\cos n\pi r}{r} - \frac{\sin n\pi r}{r}\right)\Big|_{r=1} = \sqrt{2}n\pi(-1)^n$$

This comes from your derivative of your basis equation. If you don't end up with this in your transform integration by parts then you did something wrong.

20 Question 6-2ab

For part a) table A1 is your best friend. Don't forget to split the material derivative into it's two components. $v \cdot \nabla v$ can be simplified by Table A1-11 with a and b being v. You should not have to actually evaluate the curl at any point.

b) This is pretty darn straightforward. You need to prove that $w \cdot \nabla v$ is 0. The easiest way is to just multiply out ∇v and cancel where appropriate from what you know about planar flow and the vorticity.

21 Question 6-3

a) Use the continuity equation and integrate the differential equation it creates.

b) Use what you learned from part a and the definition of vorticity. There should be two values of m which will work.

c)Do not find the values of m. Find P(x,y) assuming that m=1. You will have to recover the pressure from the gradient of the velocity. This is sometimes known as partial integration. You probably covered this in a math class a while ago.

22 Question 6-8

If you understand how to work with stream functions, this should be straightforward.

23 Question 7-1

This is a pretty simple problem that is mostly testing your understanding of the concepts taught in the chapter. All of the tools are there to solve this and there are multiple ways to do it. Don't overthink things

24 Question 7-11

You will need to do an FFT for this problem. Use figure 7-1. The side length is 2H so the center is at (H,H). Think carefully about your boundary conditions. You can treat $d\mathscr{P}/dx$ as a constant throughout the problem.

25 Question 7-14

This problem is tricky. The easiest way to define the system is with the cylinder at 0 and the wall at H on the y axis with the velocity in the positive x direction. This does tricky things to your force directions so be careful to keep track of things. Two diagrams are helpful. One of the full cylinder and one of the local region to the side of the cylinder. To solve this there are several steps:

1. Find v_x in terms of y. This should have 3 terms of y. You cannot cancel $d\mathscr{P}/dx$ so make sure to keep it.
2. The rate of volume displacement by the cylinder = the Flow rate through the gap. In math this means that
$$A_{cylinderbottom}U = 2\pi R_i \int_0^H v_x dy$$
This will allow you to solve for $d\mathscr{P}/dx$ and remove it from your v_x equation.
3. Do a force balance on the cylinder. This will involve the buoyant force, the dynamic pressure force and the viscous force. Pay very careful attention to your coordinate system and the direction of these forces.
4. solve for μ

26 Question SP12-1

Follow the example 8.4-2. Make sure to read the actual text of the example, especially pg 328 where they give the equation they got for the pressure. This will give you directions and hints as to how to solve for the pressure and eventually get the drag force.

27 Question 8-4

Part a: prove that $\nabla \cdot v = 0$ For part b When it says that $v_\phi = rf(\theta)$ you should literally just substitute v_ϕ in stokes equation. For the boundary conditions, both are no slip. Be careful of your coordinate system and think carefully about the fact that the cone is moving. What would the ϕ component of that motion be?

For part c things get a bit trickier. The best way to deal with all of the sin and cos is to do a Taylor series expansion on $\sin(\theta)$ and $\cos(\theta)$ about $\frac{\pi}{2}$ (only worry about the first 2 terms). This will clean up the differential equation immensely. Additionally. $\frac{\pi}{2} - \theta \leq \beta$ Since $\beta << 1$ you can cancel the term that has that multiplied to it. The final equation you get should be very easy to solve.

For part d start from $dG = r|\tau_{\phi\theta}|_{\theta=\pi/2}dS$ where $dS = rdrd\phi$