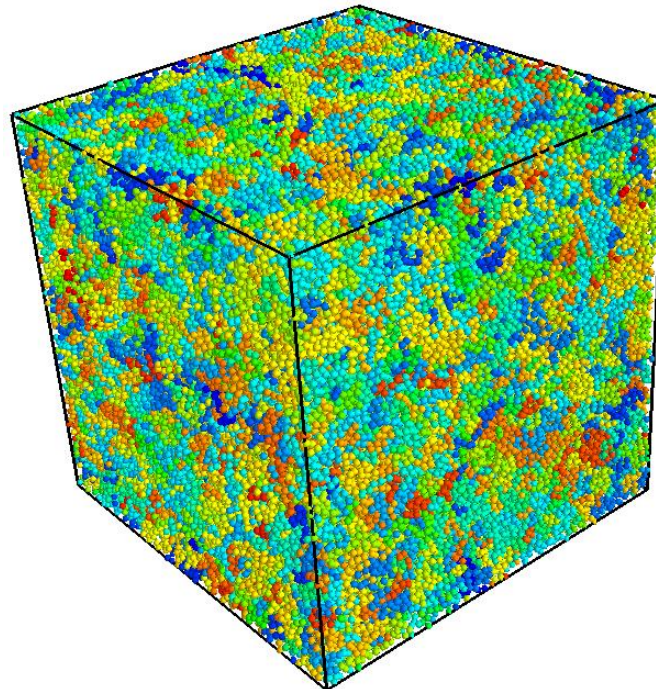




# Wang-Landau Simulation of the Free Energy Surface of Crystallization in a Polymer Melt



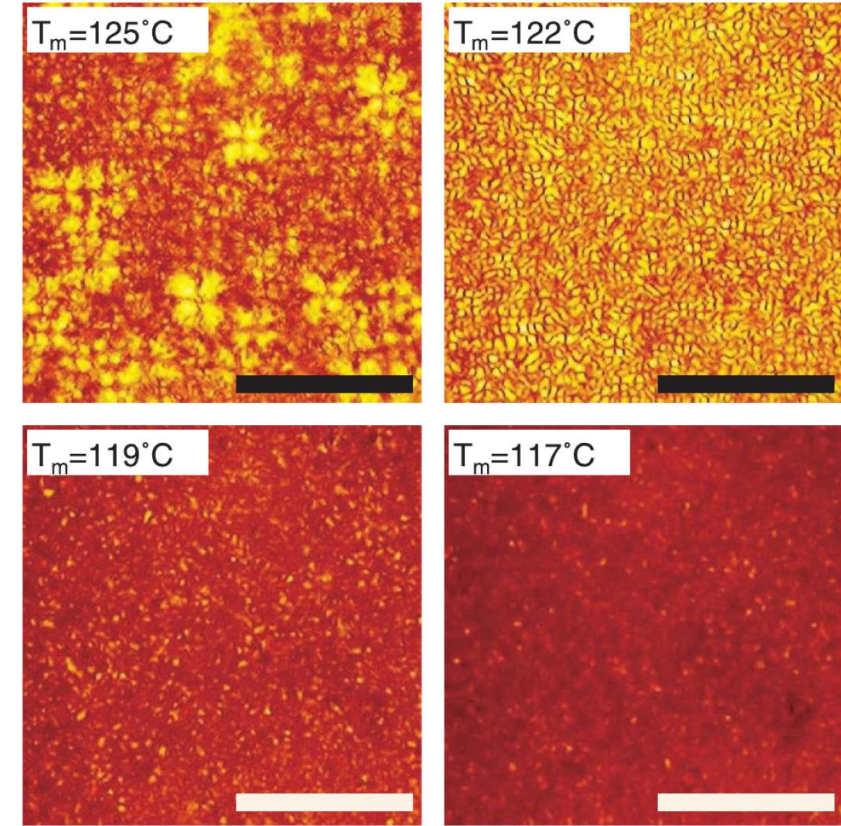
Pierre Kawak, Andrew S. Gibson, Logan S. Brown, Beverly Delgado, Douglas R. Tree



# There is a controversy in the polymer crystallization literature

## Melt Memory Effect & Primary Nucleation

- Häfele et al. *Eur Phys J E* (2005)
- Final crystalline state depends on  $T_m$
- Influenced by slow dynamics or thermodynamic phenomena?



## Other Observations:

Initial larger scale ordering in SAXS and WAXS

*Imai et al. Physica B Condens* (1995)

Deviant Crystallization and Recrystallization

*Strobl Rev Mod Physics* (2009)

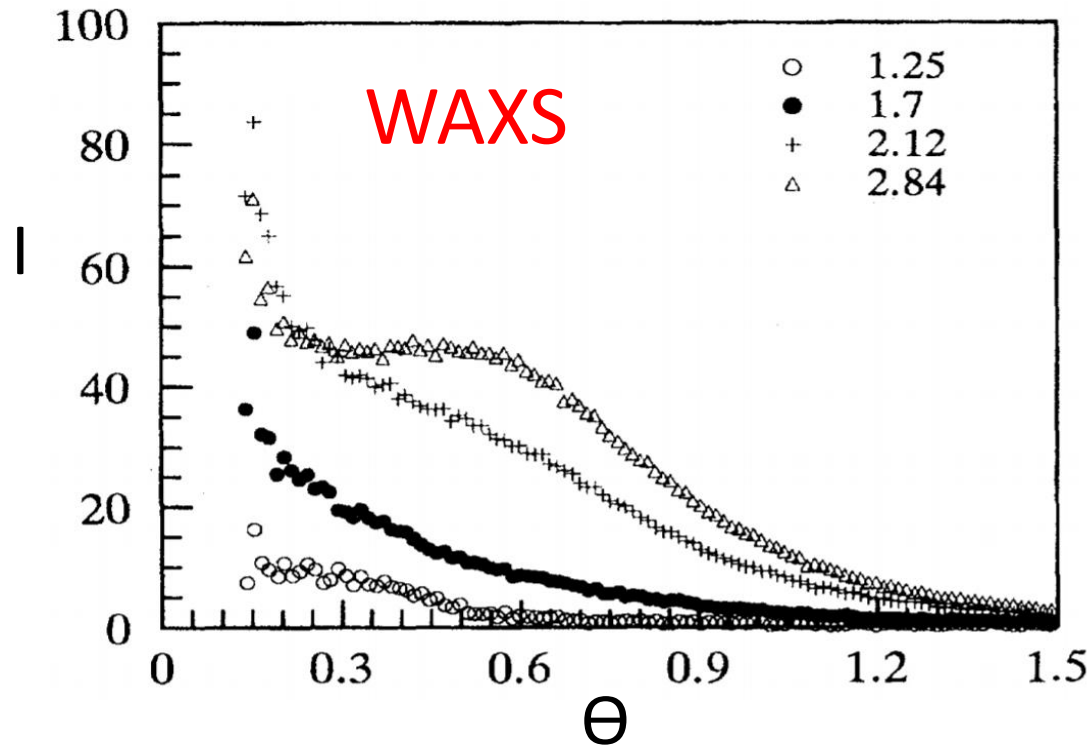
Intermediate Phase Observations *Kanig Colloid Polym Sci* (1991), *Rastogi et al. Macromol* (1991)

No Copolymer Effect on Crystallization

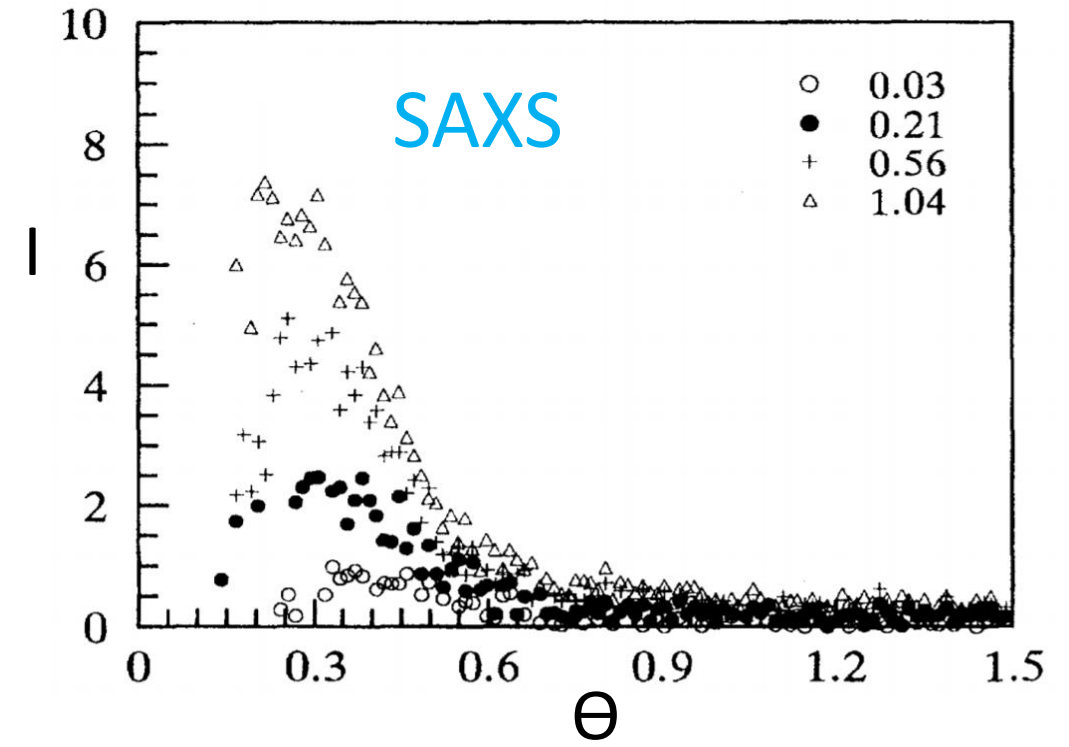
*Hauser et al. Macromol* (1998)

# Small- and Wide- Angle X-ray Scattering reveals initial ordering prior to crystallization

Imai et. al, 1995, Phys. Rev. B, 52



- Probes Å-scale behavior
- Order at  $t = 2.84$



- Probes nm-scale behavior
- Order at  $t = 1.04$

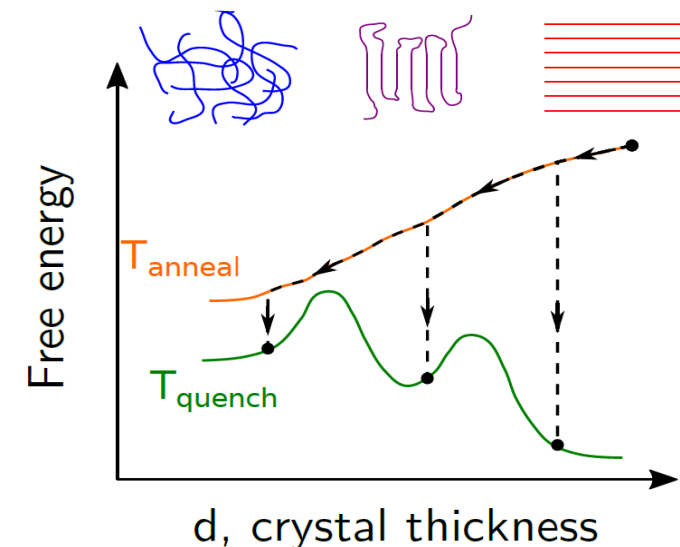
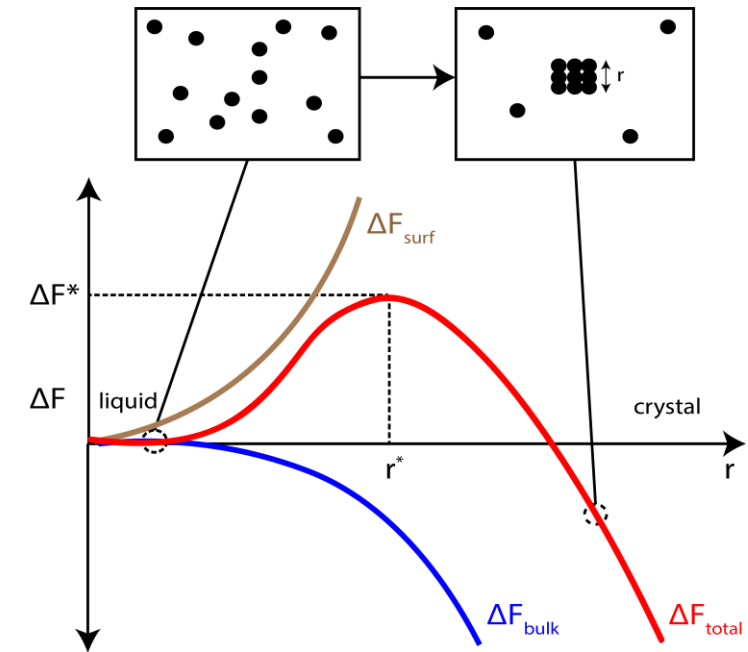
# Can the previous observations be explained by CNT?

## Classical Nucleation Theory (CNT)

- Single energy barrier predicted
- No intermediate mesophases
- Could more than one barrier explain prior observations?

## Current Hypothesis

- Strobl, Olmsted, Milner and Muthukumar (SOMM) propose intermediate nematic phase
- SOMM is controversial; some authors believe slow entanglement dynamics cause these observations



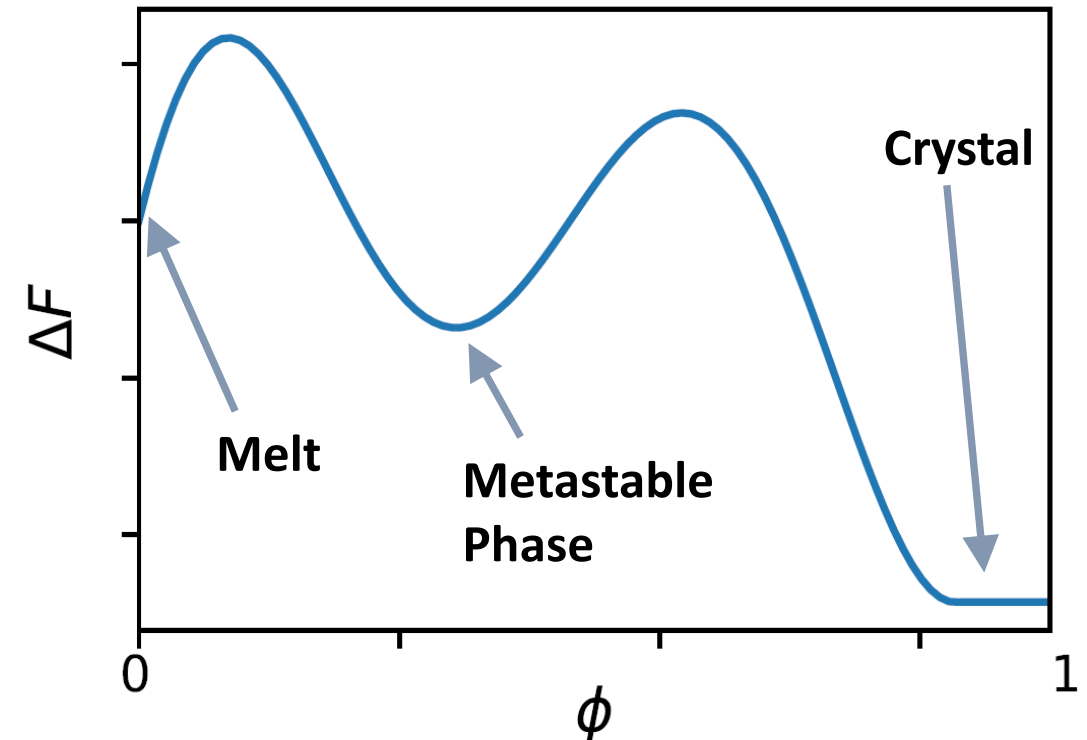
# The literature and our approach

## Current State of the Literature

- Very smart people are trying: Sommer, Rutledge, Milner, Hoy, Yamamoto, Sirota, Zhang, Paul, etc.
- We have an interesting approach; build a Free Energy Surface (FES) for:
  - Appropriate system sizes
  - Homogeneous nucleation from amorphous melt

## Advantages of FES

- Direct test of CNT/SOMM
- Equilibrium calculation
- No dynamic considerations
- $F(\phi_1, \phi_2, \phi_3, \dots, \phi_n)$



# What simulation techniques can build an FES?

## Molecular Dynamics (MD)

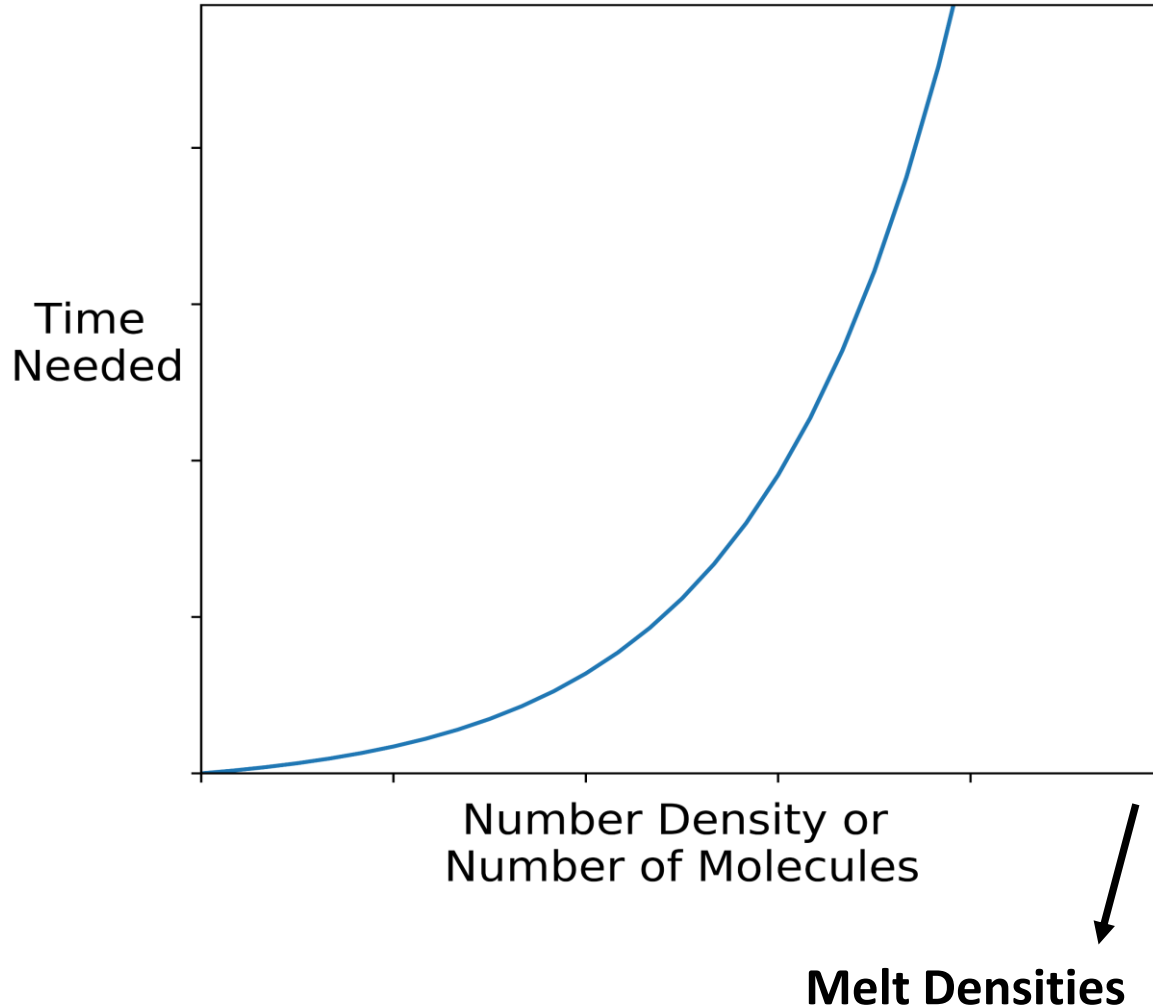
- Access to dynamic properties
- Readily available (e.g. LAMMPS)
- Limited by entanglement dynamics
- Very slow equilibration

## Wang-Landau Monte Carlo (WLMC)

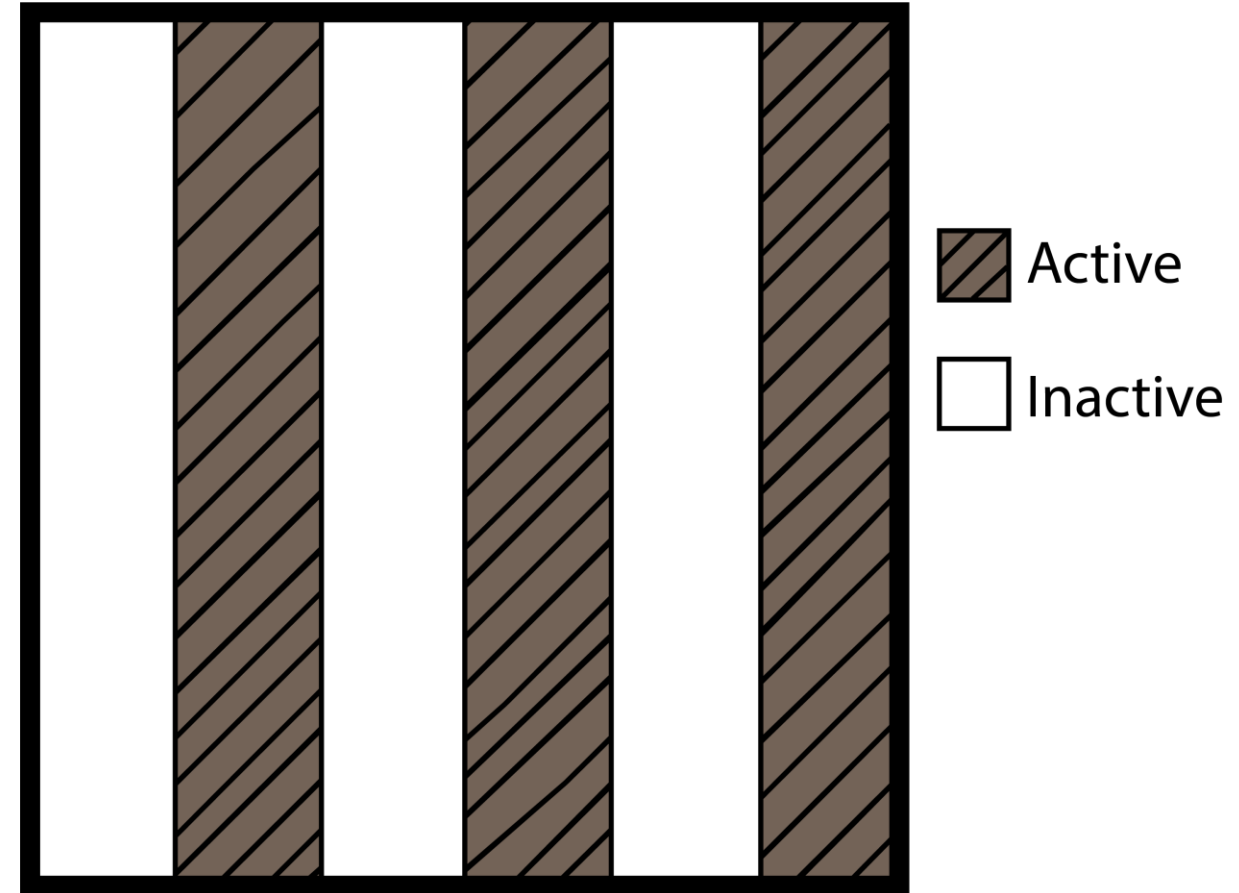
- Bond-breaking moves
- Overcomes energy barriers
- Direct density of states ( $\Omega$ ) computation
- That is, direct FES computation

# Parallel simulations reduce required simulation times

Polymer simulations are expensive

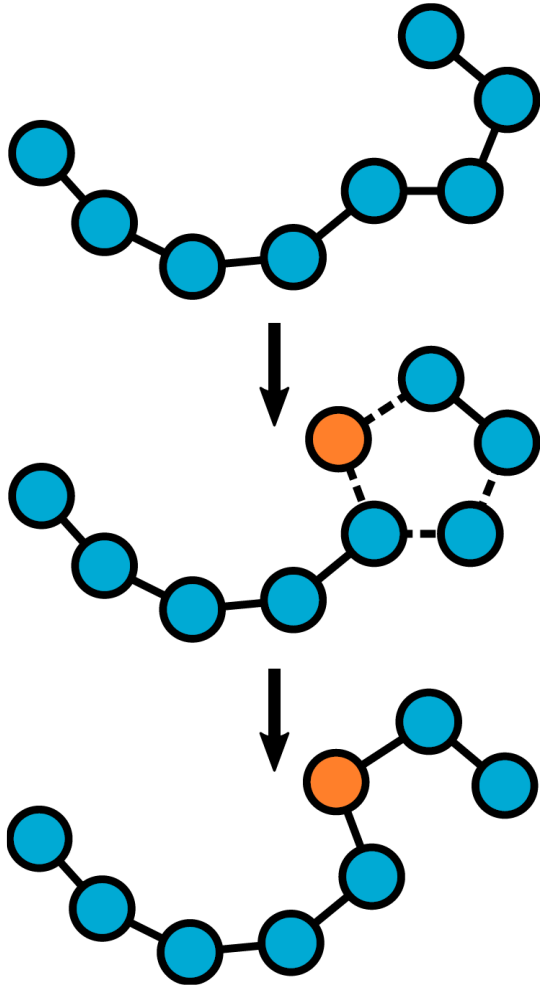


Domain decomposition parallel construct

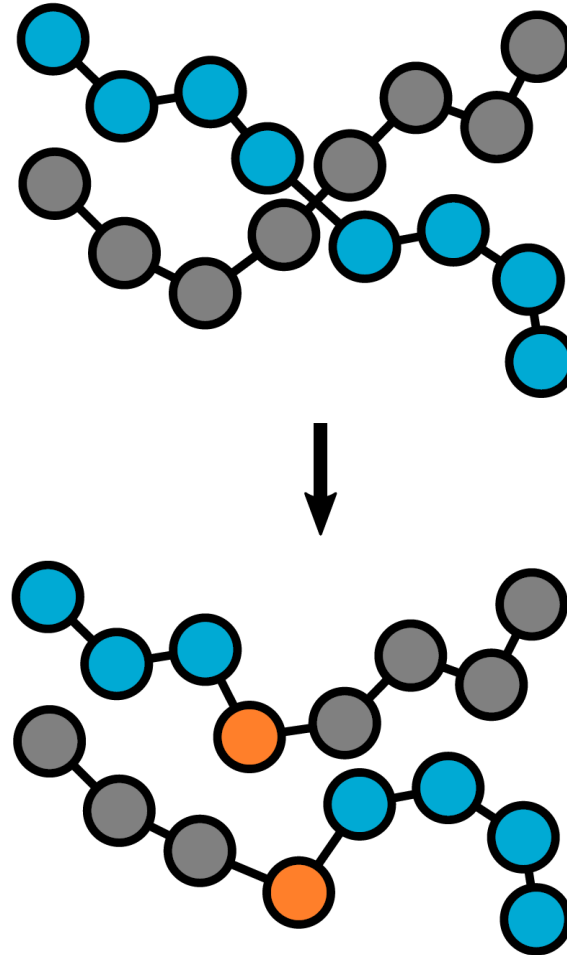


# Advanced moves help break entanglement dynamics

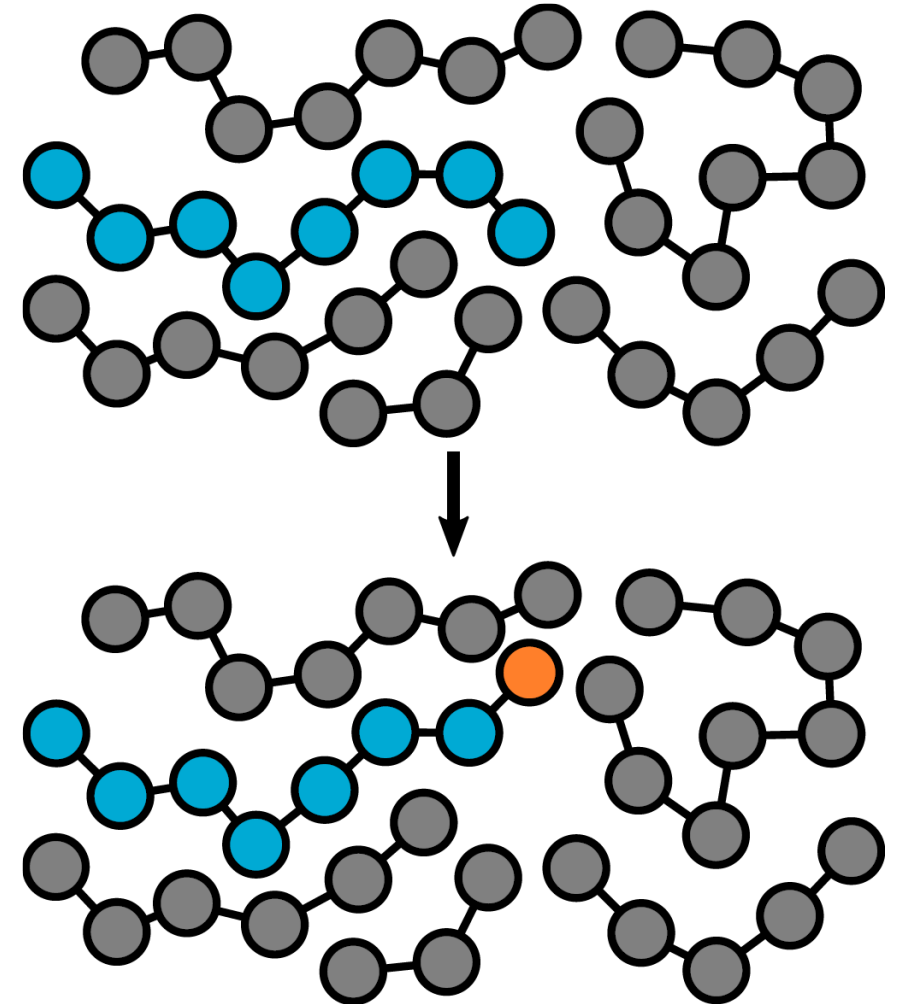
End-bridging



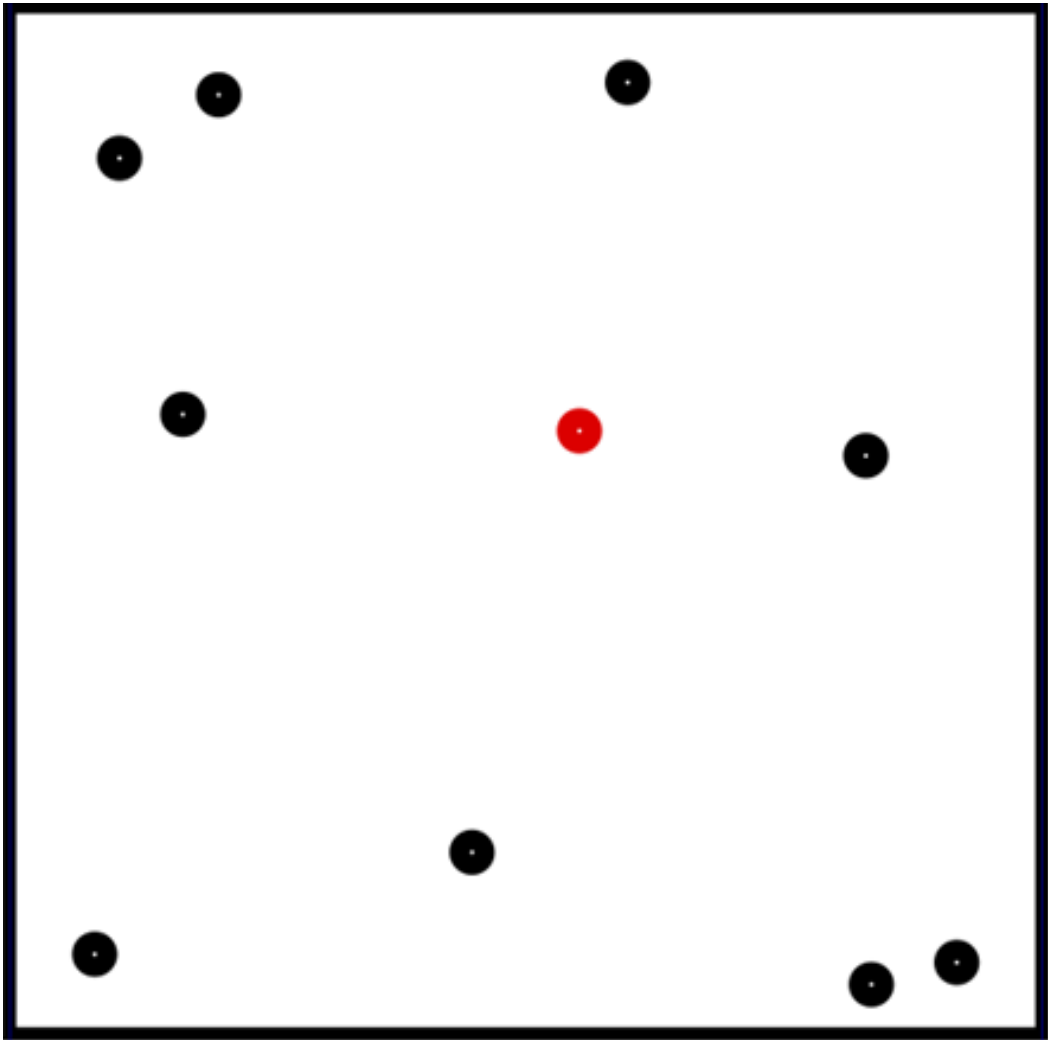
Double-bridging



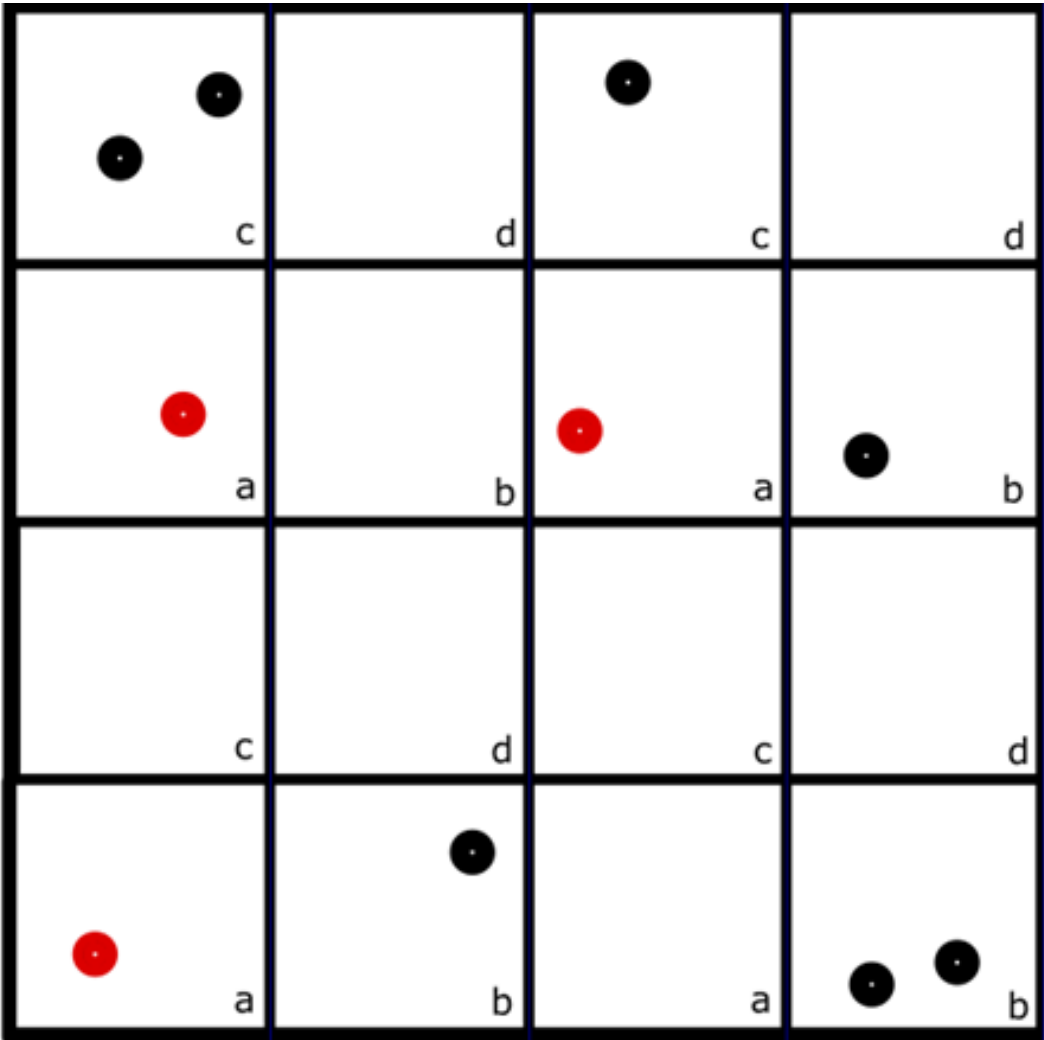
Configurational Bias



# Domain decomposition breaks up the simulation volume



Conventional MC



Checkerboard MC

Anderson et al. J. Comp. Phys. (2013)  
Anderson et al. Comp. Phys. Comm. (2016)

# A WLMC-generated FES can characterize crystallization

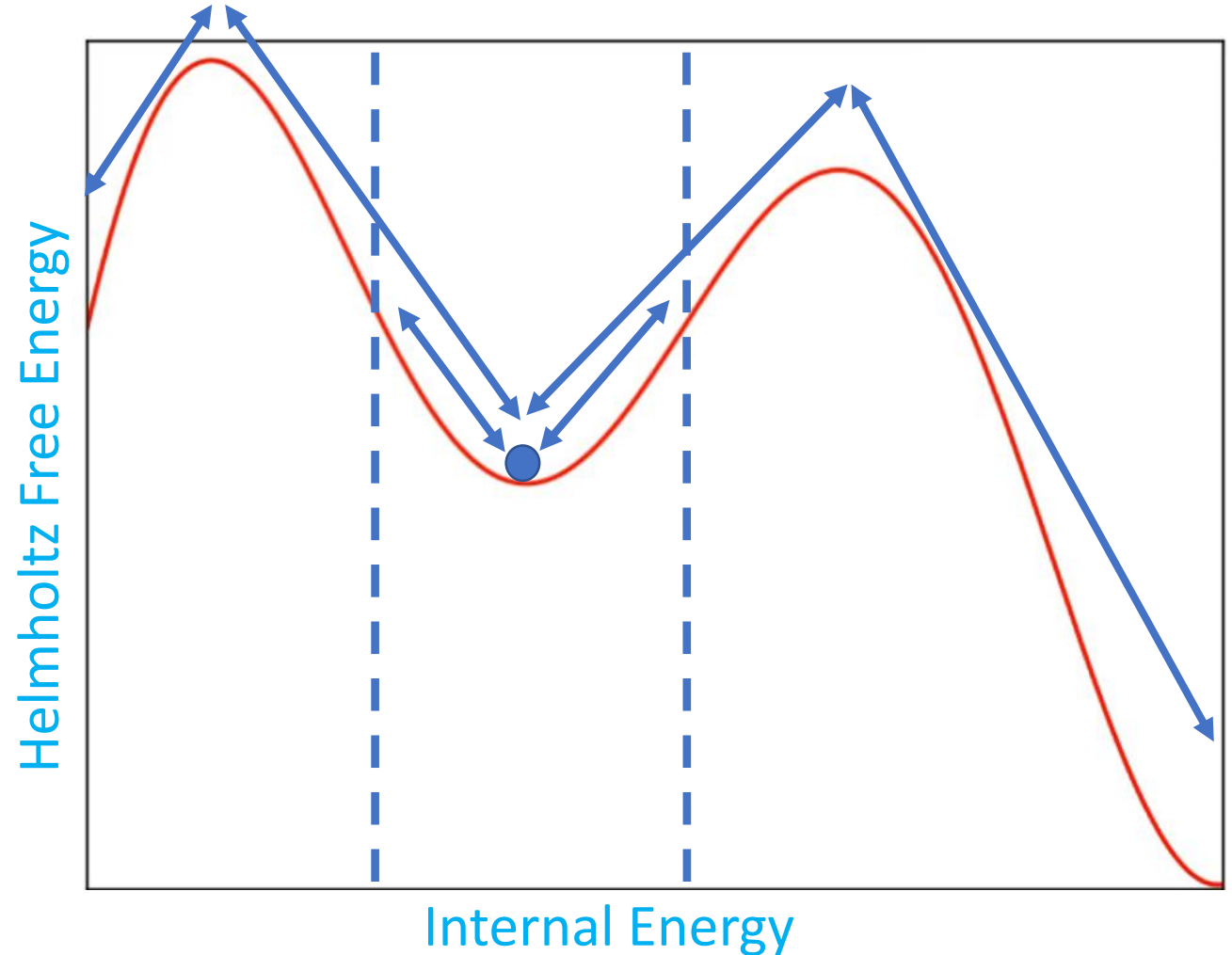
Metropolis MC:

$$P = e^{-\Delta E/T}$$

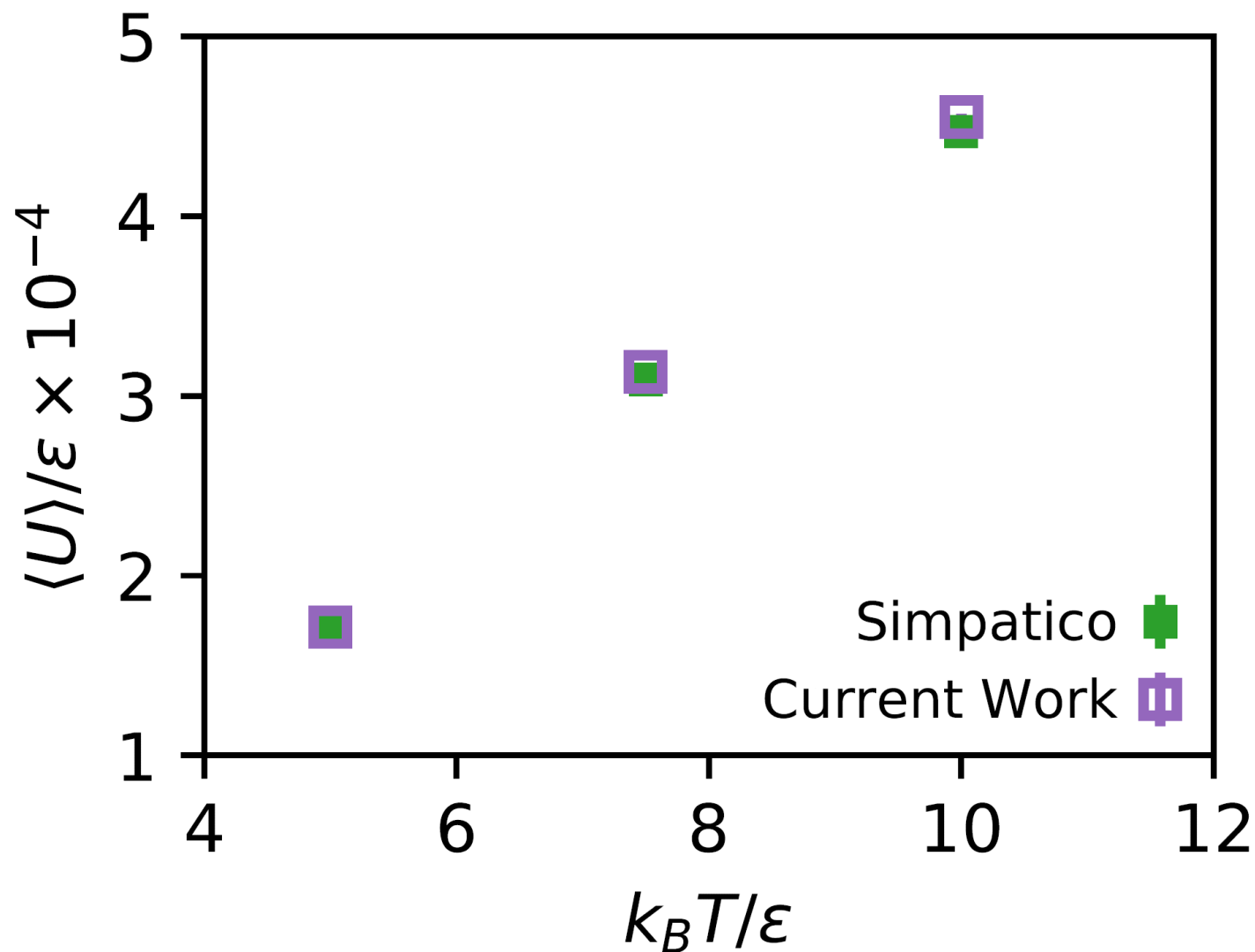
Wang-Landau MC:

$$P = e^{\Omega(E_{old}) - \Omega(E_{new})}$$

- Better sampling of phase space
- Overcomes energy barriers
- Direct access to  $\Omega$  (DOS)



# Verification of our results with Simpatico



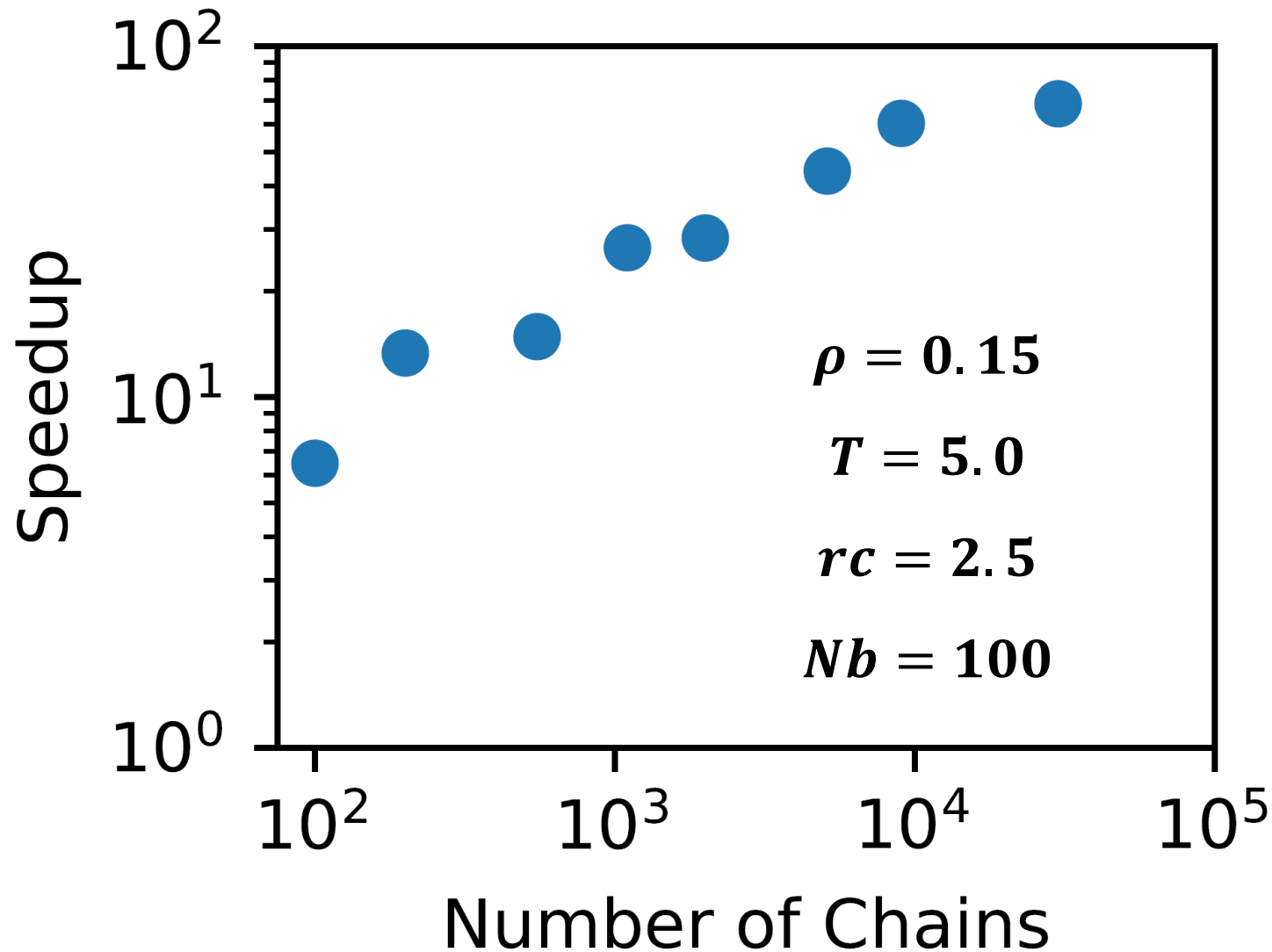
*LJ Bead-Spring System*

$N_{chains} = 100$

$N_{beads} = 100$

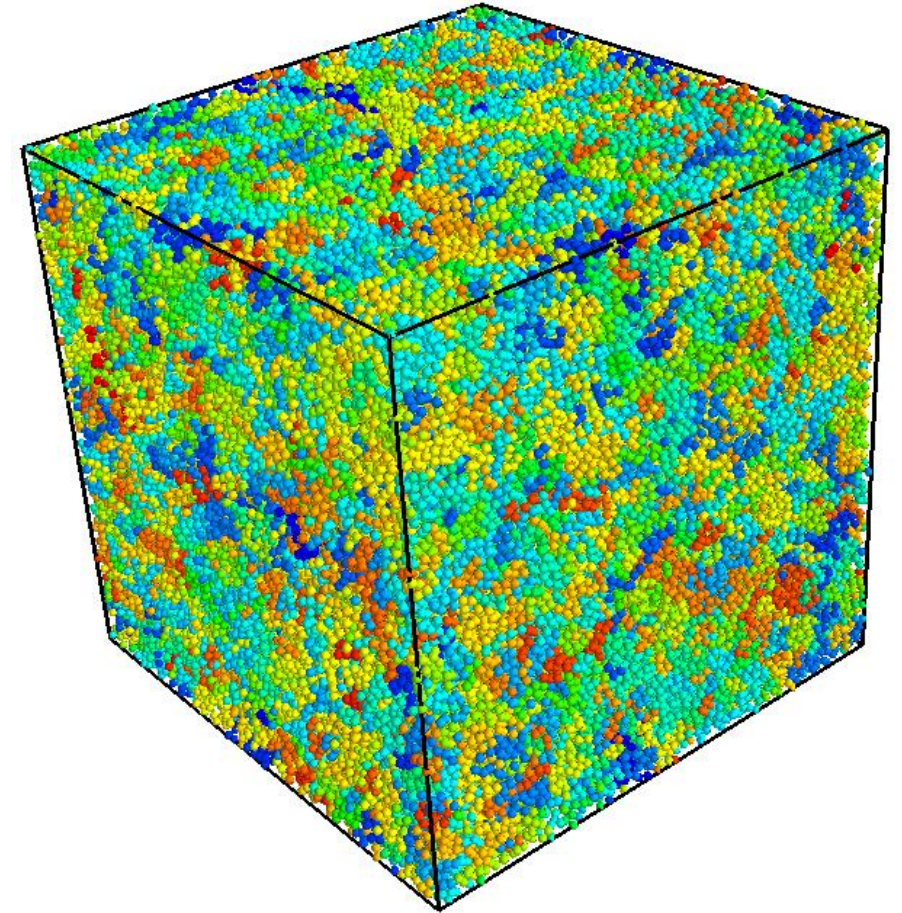
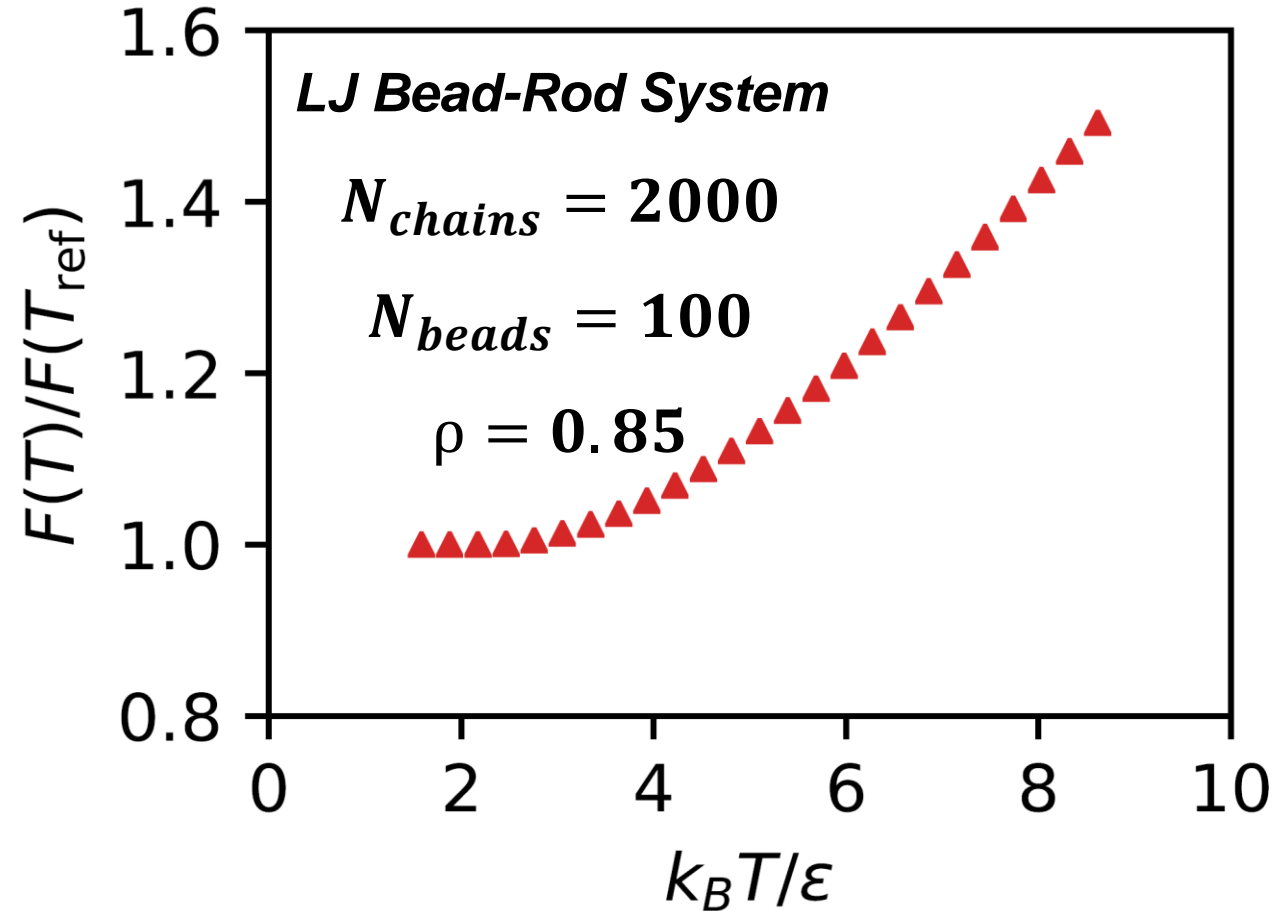
$\rho = 0.15$

Parallel simulations are 2 orders of magnitude faster



$$\text{Speedup} = \frac{\text{time per move}_{\text{serial}}}{\text{time per move}_{\text{parallel}}}$$

# WLMC simulation of bead-rods yields single transition FES



Results from bead-rod simulations

- Consistently with Hoy et al.'s work, single transition was observed (RCP)
- CNT is an appropriate description for simple bead-rod systems

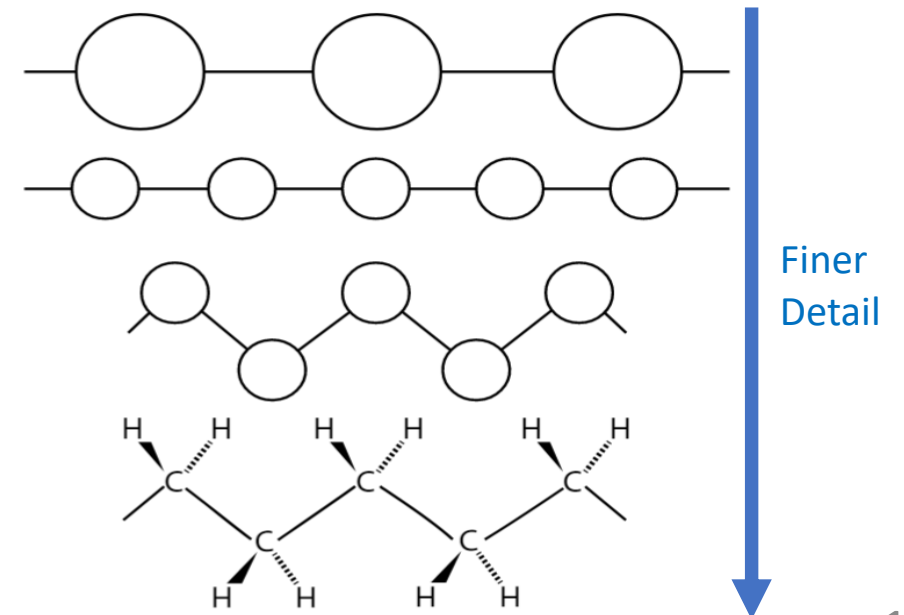
# Conclusion and Future Work

## Current Work

- We built a **GPU-accelerated WLMC** algorithm from scratch
- CNT is an appropriate description for simple bead-rod systems
- Chain connectivity not enough to capture important polymer phenomena

## Future Work

- Explore different **order parameters**
- More realistic detail
  - **Semi-flexible** chains
  - Torsion potentials
- **Long chain** studies capturing entanglement



# Acknowledgements

- ACS Petroleum Research Fund
- BYU Board of Trustees
- BYU Office of Research  
Computing and Fulton  
Supercomputing Lab

