



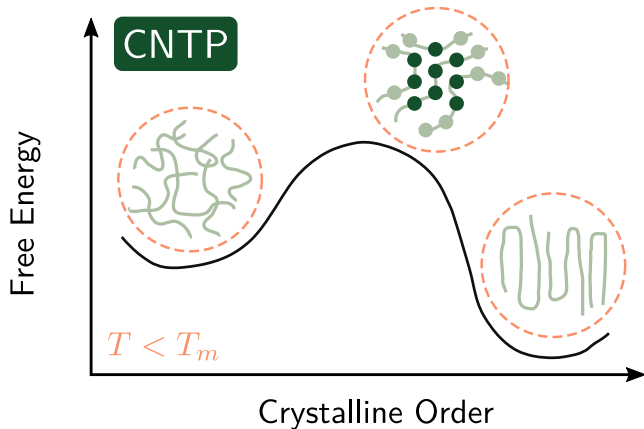
Free Energy Surfaces for Homogeneous Nucleation in a Polymer Melt

Douglas R. Tree, Pierre Kawak, Dakota S. Banks

Brigham Young University

How does a polymer crystal nucleate from a melt?

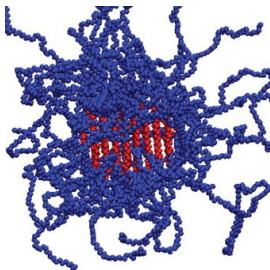
Classical Nucleation Theory for Polymers



Schultz. Polymer Crystallization. (2001)

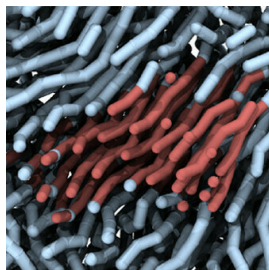
Yi. J. Chem. Phys. (2009)

Controversy in non-equilibrium MD simulations of polyethylene



- ▶ Cylindrical crystal nucleus
- ▶ Crystal size is a good order parameter
- ▶ No intermediate phases/states
- ▶ General agreement with classical models

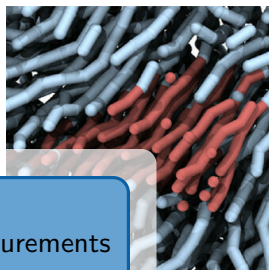
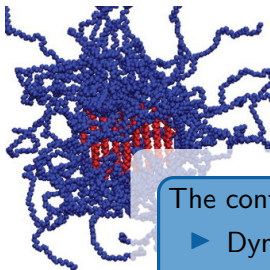
Yi et al. Macromolecules (2013)



- ▶ Anisotropic crystal nucleus
- ▶ Crystal size is not only order parameter
- ▶ Crystal nucleates inside nematic droplet
- ▶ Disagrees with classical models

Hall et al. J. Phys. Chem. B. (2020)

Controversy in non-equilibrium MD simulations of polyethylene



The controversy extends to:

- ▶ Dynamic SAXS/WAXS measurements
- ▶ Recrystallization and melt memory experiments

- ▶ Cylindrical crystal nucleus
- ▶ Crystal size is a good order parameter
- ▶ No intermediate phases/states
- ▶ General agreement with classical models

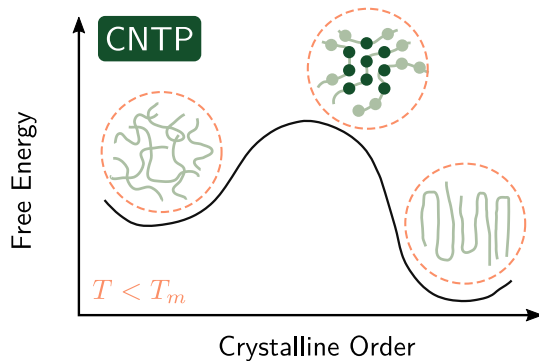
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Hall et al. J. Phys. Chem. B. (2020)

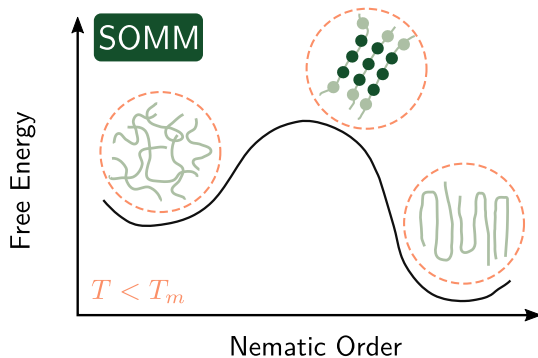
Contradictions in simulations and experiments have led to new theories

Classical Nucleation Theory for Polymers



Schultz. Polymer Crystallization. (2001)
Yi et al. J. Chem. Phys. (2009)

"SOMM" Hypothesis

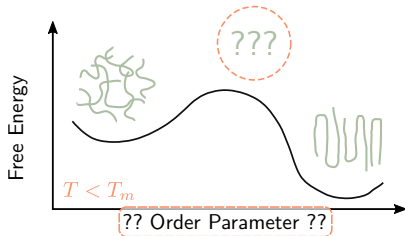


Strobl. The Physics of Polymers. (2007)
Olmsted. Phys. Rev. Lett. (1998)
Milner. Soft Matter. (2011)
Welch and Muthukumar. Phys. Rev. Lett. (2001)

A way forward? Calculate the free energy landscape (FEL)

Key idea

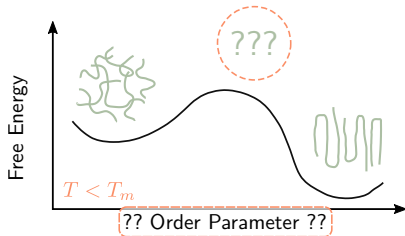
- ▶ All theories are based on a free energy landscape
- ▶ An FEL is a near-equilibrium concept



A way forward? Calculate the free energy landscape (FEL)

Key idea

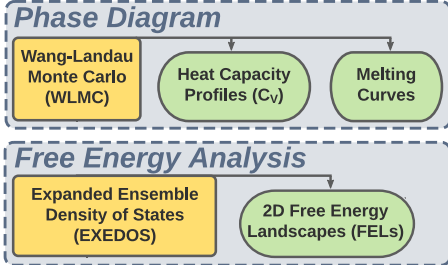
- ▶ All theories are based on a free energy landscape
- ▶ An FEL is a near-equilibrium concept



Our Approach

Use equilibrium Monte Carlo methods

- ▶ Construct a phase diagram (WLMC)
- ▶ Calculate FELs (EXEDOS)
- ▶ Study crystalline (Q_6) and nematic (P_2) order parameters



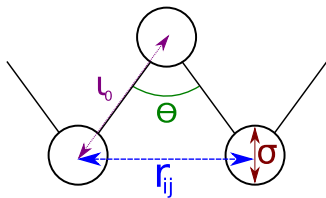
We have started with a simple model of semiflexible oligomers

$$U_{tot} = U_{hs}(r_{ij}) + U_{bond}(l) + U_{bend}(\theta)$$

$$U_{hs} = \begin{cases} \infty & r_{ij} < \sigma \\ 0 & r_{ij} \geq \sigma \end{cases}$$

$$U_{bond} = \begin{cases} 0 & l = l_0 \\ \infty & l \neq l_0 \end{cases}$$

$$U_{bend} = \begin{cases} -\epsilon & \theta \leq \theta_s \\ 0 & \theta > \theta_s \end{cases}$$



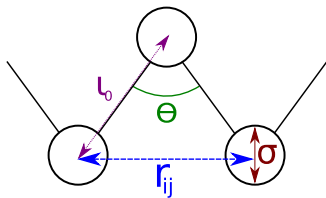
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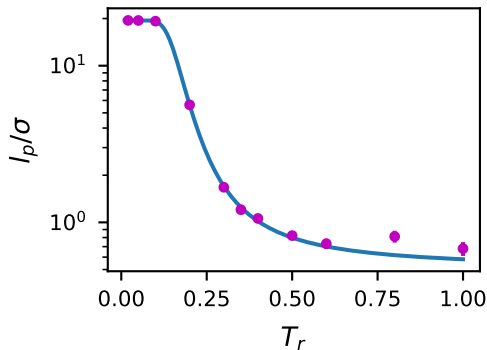
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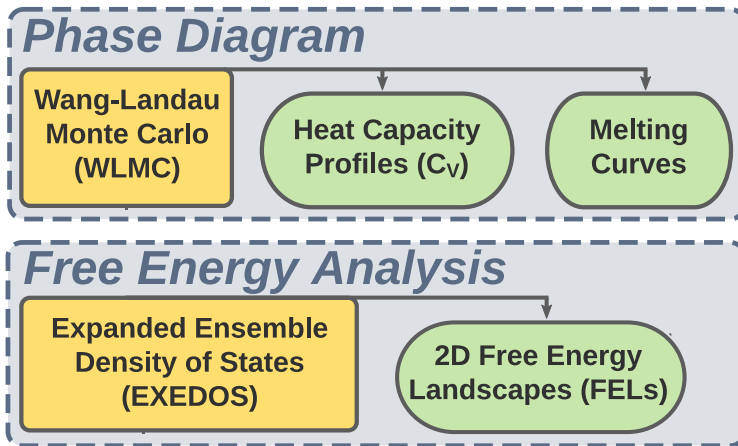


2 Degrees of Freedom

- ▶ Volume Fraction ϕ
- ▶ Reduced Temperature $T_r = kT/\epsilon$ (equivalent to persistence length l_p)



Overview

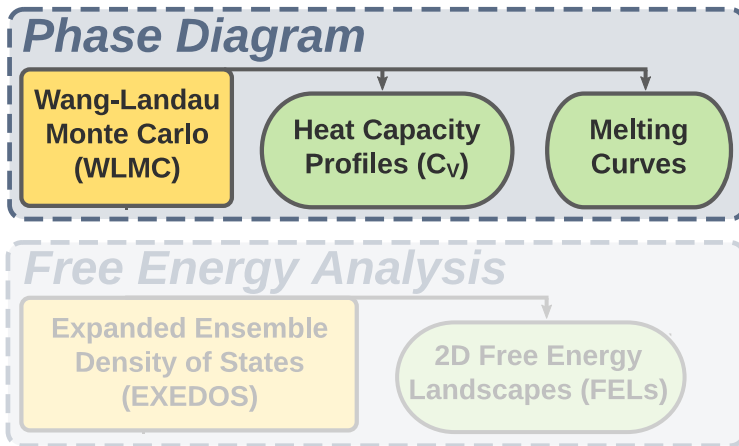


WLMC: Wang. Phys. Rev. Lett. (2001)

EXEDOS: Rathore et al. J. Chem. Phys. (2004)

Kawak, Banks, & Tree. J. Chem. Phys. (In Press)

Overview

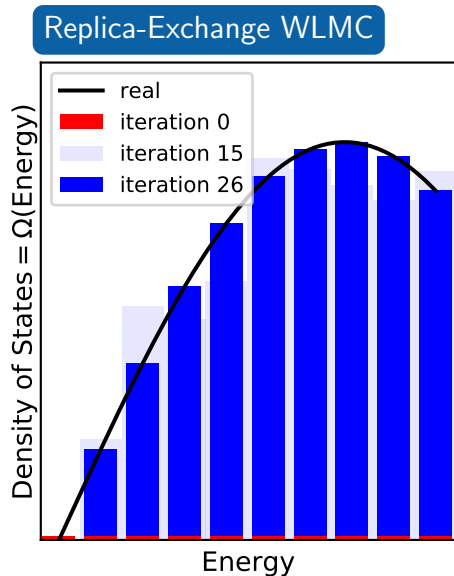


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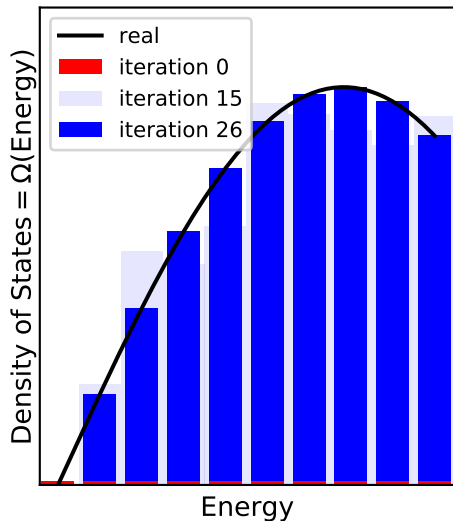
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Wang–Landau captures the T -dependent thermodynamics at fixed ϕ

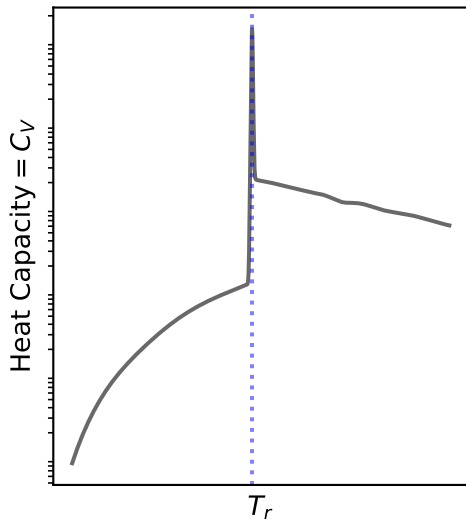


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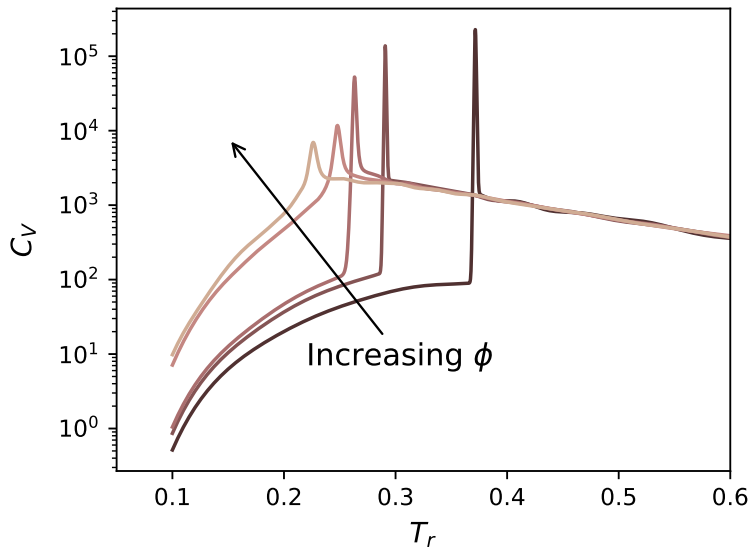
Replica-Exchange WLMC



Heat Capacity for all T_r

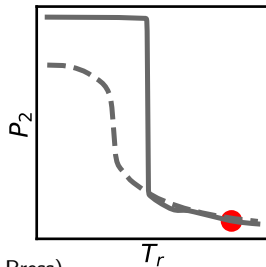
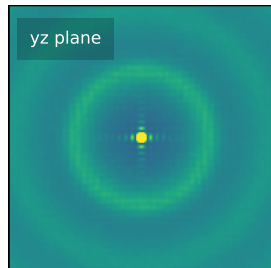
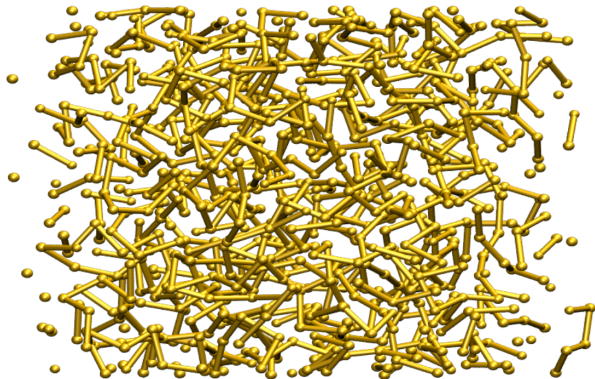


Sweeping volume fraction (ϕ) shows a single first-order phase transition



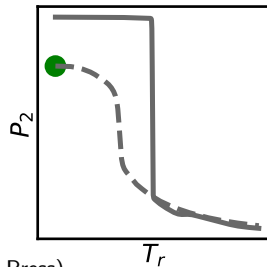
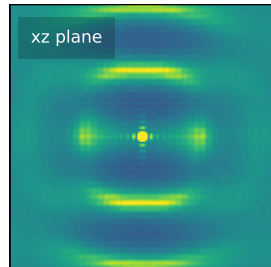
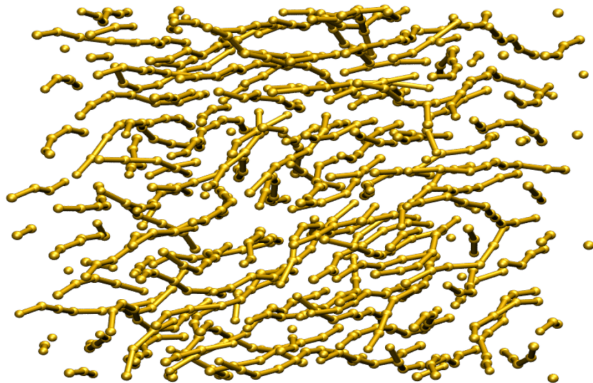
Kawak, Banks, & Tree. J. Chem. Phys. (In Press)

At high T and all ϕ the system is an isotropic melt



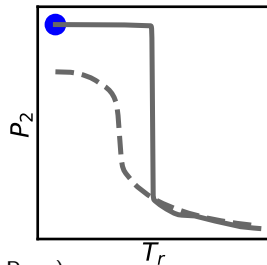
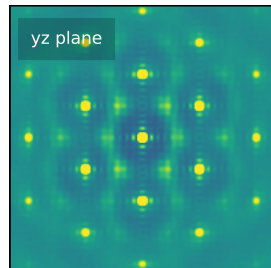
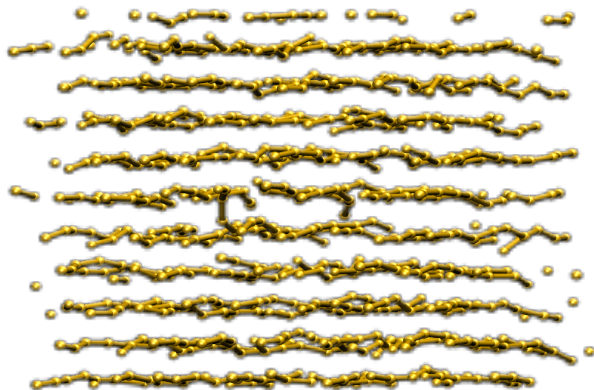
Kawak, Banks, & Tree. J. Chem. Phys. (In Press)

At low T and low ϕ the system is nematic



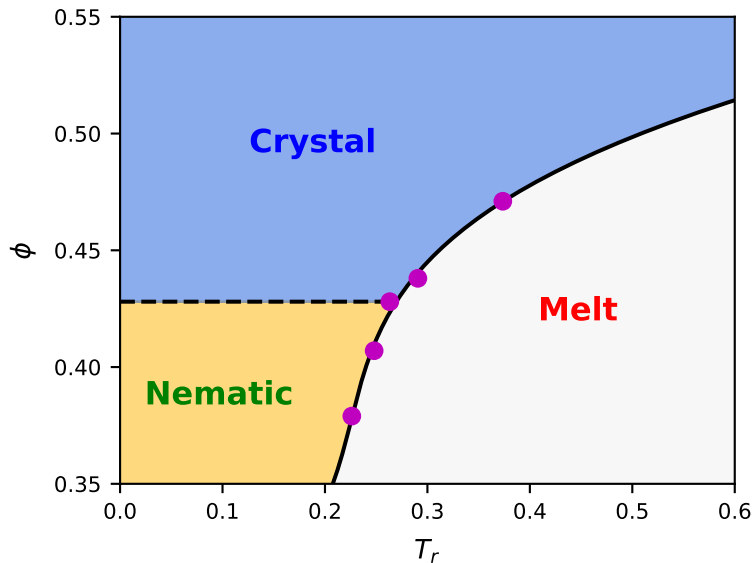
Kawak, Banks, & Tree. J. Chem. Phys. (In Press)

At low T and high ϕ the system is crystalline

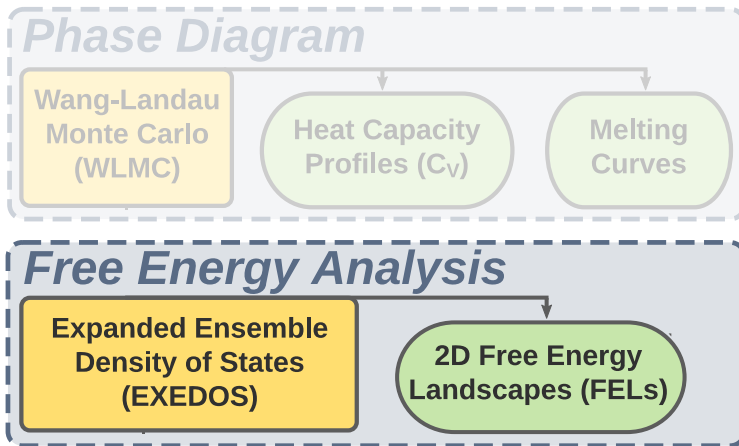


Kawak, Banks, & Tree. J. Chem. Phys. (In Press)

Melt–nematic–crystal phase diagram



Overview

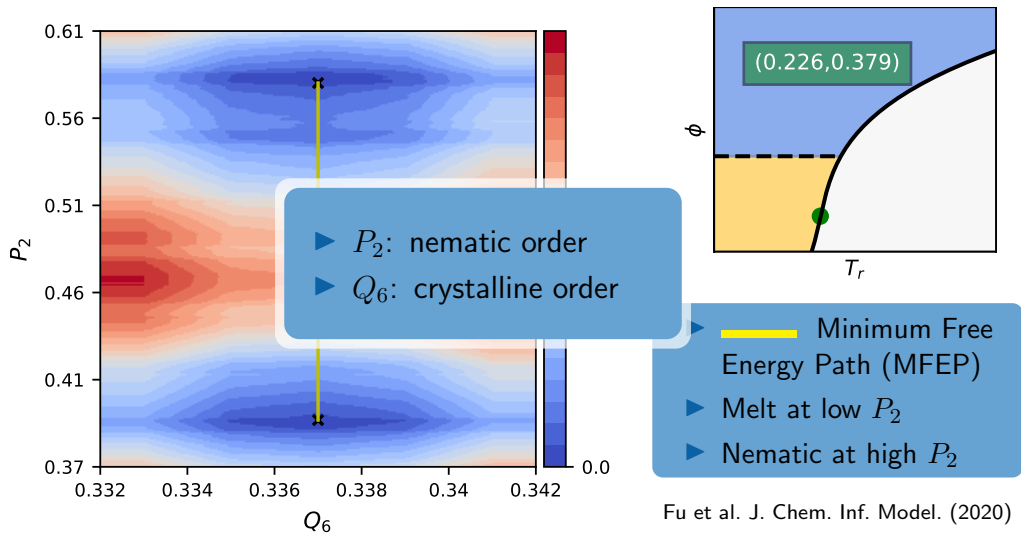


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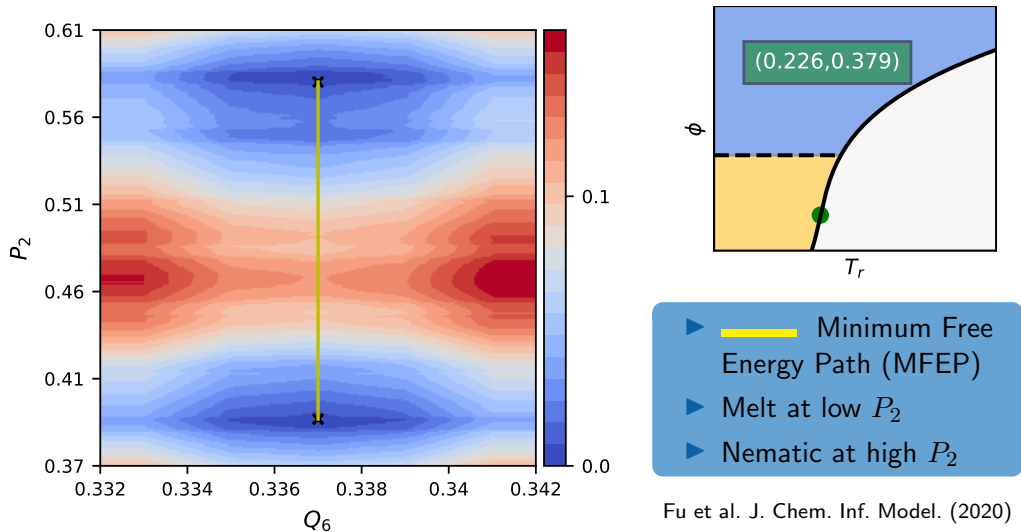
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The FEL of the melt–nematic transition is along P_2



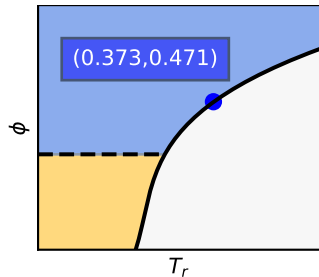
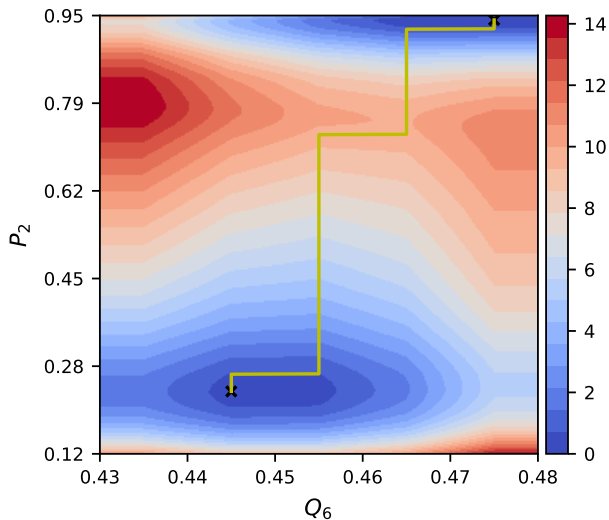
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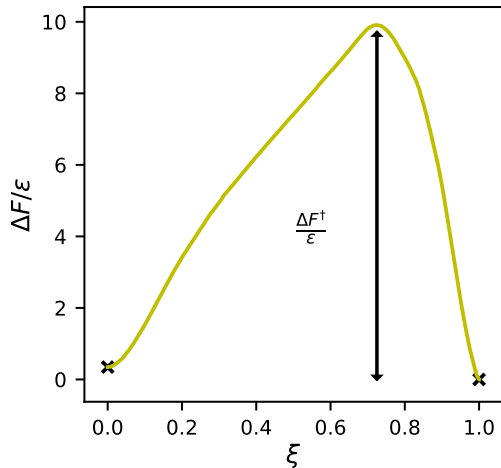
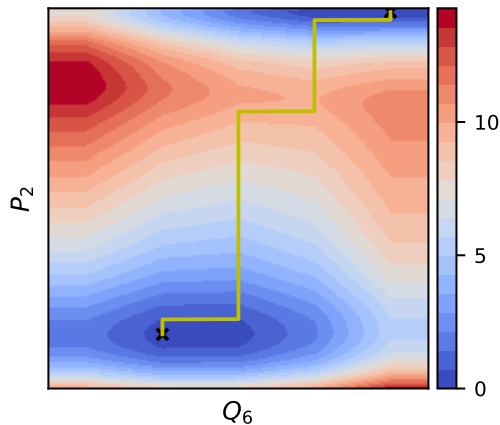
Kawak, Banks, & Tree. J. Chem. Phys. (In Press)

The FEL of the melt–crystal transition is cooperative between P_2 and Q_6

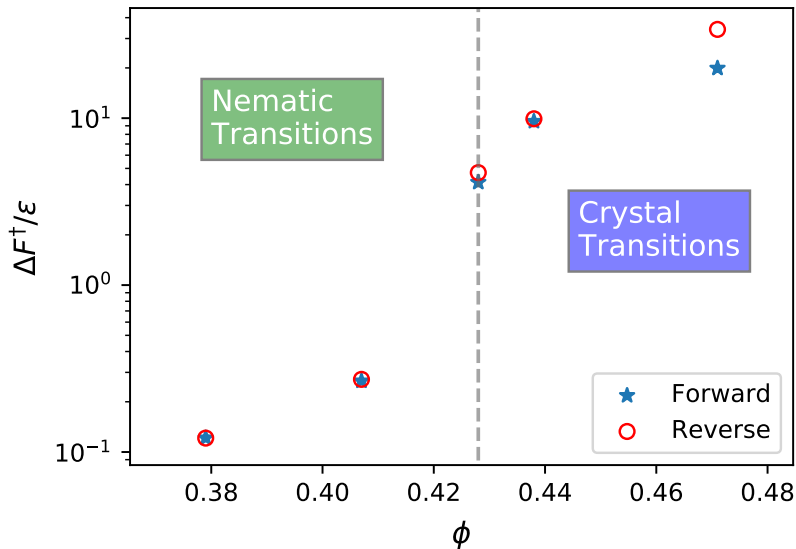


- ▶ MFEP
- ▶ Melt at low P_2, Q_6
- ▶ Crystal at high P_2, Q_6

Extracting the barrier height $\Delta F^\ddagger$ from the MFEP



The barrier height $\Delta F^\ddagger$ is strong function of ϕ



Conclusion: “This work opens more questions than it answers”

Conclusions

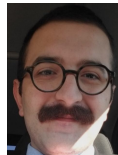
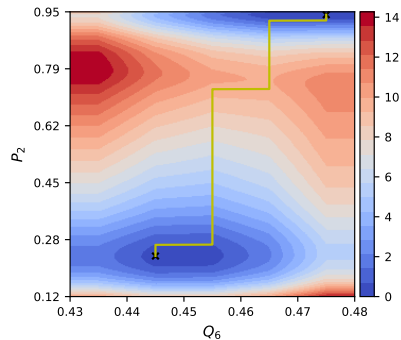
For a system of simple semiflexible oligomers:

- ▶ Equilibrium FELs are valuable
- ▶ No intermediate between melt and crystal
- ▶ Nematic and positional ordering cooperate in crystallization mechanism

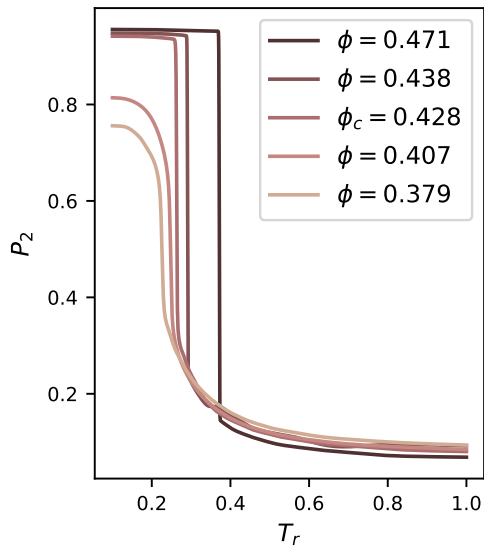
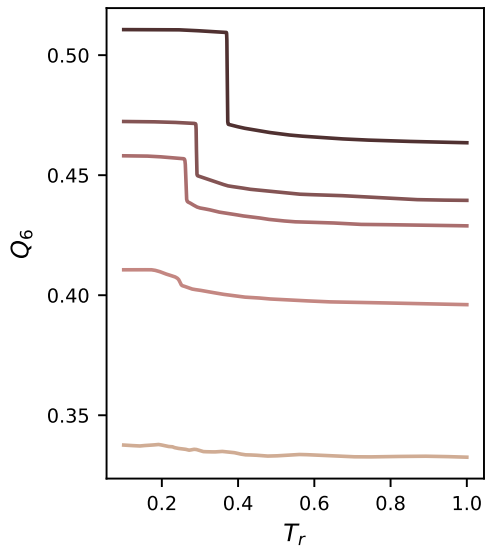
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How general are our conclusions?

- ▶ Molecular weight?
- ▶ More complex intermolecular potentials?



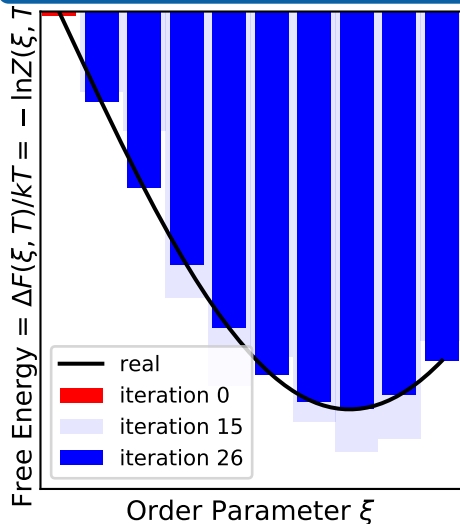
Order parameters reveal two types of transitions



Kawak, Banks, & Tree. J. Chem. Phys. (In Press)

To understand interplay of Q_6 and P_2 , we need 2D FELs

EXEDOS algorithm builds 1D FEL



Our algorithm incorporates 2 OPs

