

Solutions to Some Problems, Calculus (Third. Ed.)

Chapter 1

Section 1.1

1. When they overlap or when they do not overlap but share an endpoint that belongs to one of the intervals. No.

4. (a)(ii) If $c \neq 0$, then $\frac{1}{c}$ exists. Hence $a = a \cdot 1 = a(c \cdot \frac{1}{c}) = (ac) \cdot \frac{1}{c} = (bc) \cdot \frac{1}{c} = b(c \cdot \frac{1}{c}) = b \cdot 1 = b$. (b)(ii) $a < b \Rightarrow a + c < b + c$ and $c < d \Rightarrow b + c < b + d$. Hence by transitivity [6b, p. 4], $a + c < b + d$. (c) $a \cdot 0 = a(0 + 0) = a \cdot 0 + a \cdot 0 \Rightarrow 0 = a \cdot 0$ by 4(a)(i). (d)(ii) Let $c = b - a$. (e) (i) $0 = 1 \Rightarrow a = a \cdot 1 = a \cdot 0 = 0$ for every a . Since $\mathbb{R}$ has more than one element, we must conclude that $0 \neq 1$. (ii) $1 < 0 \Rightarrow 1 + (-1) < 0 + (-1) \Rightarrow 0 < -1 \Rightarrow 0(-1) < (-1)(-1) \Rightarrow 0 < 1$, contradicting 6a, p. 4. Hence $1 < 0$ cannot hold. (iii) By 6a, p. 4, $0 < 1$ is the only possibility.

5. (b) $b < a \Rightarrow 0 < a - b$; let $c = a - b$. Then $0 < c$ and $|a - b| = |c| = c = a - b$. (d) By (b), if $x < 0$, then $|x| = |x - 0| = |0 - x| = 0 - x = -x$.

6. (c) By (b), $|x - c| < \delta \Rightarrow c - \delta < x < c + \delta \Rightarrow x \in (c - \delta, c + \delta)$.

7. (a) (1.99, 2.01). (b) (3.5, 6.5). (d) (3.999, 4.001)

9. If l_1 and l_2 are both greatest lower bounds of S , then each must be at least as large as the other, so they are equal. Thus there is only one glb.

11. (b) 1, 0. (d) 1, 0. (f) $\sqrt{2}$, 0. (h) None, 0.

13. (a) If S is finite, arrange the elements of S in increasing order. Then the first element is a lower bound and the last element is an upper bound. (b) The largest element is the lub. (c) The least element is the glb.

Section 1.2

1. (a) II. (b) III. (c) I. (d) ... stopped for a while...

2. The graph is probably increasing, leveling off into a sine wave.

6. Increasing, leveling off to the temperature of the oven, and concave downward, because its rate of change of temperature becomes smaller as the temperature gets closer to the temperature of the oven.

25. If two points (x, y_1) and (x, y_2) are on a vertical line and also on the graph, then the numbers y_1 and y_2 are both associated with x , violating the definition of function.

27. (b) $(-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty)$. (d) $(3, \infty)$. (f) $x \neq 4$.

28. (b) $(0, \infty)$. (d) $(0, \infty)$.

30. (b) $(-\frac{3}{2}, \frac{7}{2})$.

Section 1.3

2. (b) $y = 5 - x$. (d) $y = -x - 1$.

3. (a) i. (b) vi. (c) iv. (d) ii. (e) v. (f) iii.

4. (a) vi. (b) ii. (c) iv. (d) v. (e) i. (f) iii.

6. (b) $\frac{x}{-1} + \frac{y}{3} = 1$. (d) $\frac{x}{0.35} + \frac{y}{1.1} = 1$.

11. If the lines have different slopes, then they cannot be parallel, so must meet. Find the intersection point by solving the equations simultaneously.

13. By the exterior angle theorem, $\phi_2 = \phi_1 + \theta$, so that $\theta = \phi_2 - \phi_1$. Now use the tangent subtraction formula and the fact that $\tan \phi_i = m_i$.

15. Cross-multiply and solve for t .

25. $f(c) \approx y_1 + \frac{y_2 - y_1}{x_2 - x_1}(c - x_1)$. Since this approximation uses a straight-line approximation, it may not be accurate, for f may not actually be a linear function.

28. The distance from $P : (x_1, y_1)$ to $l : Ax + By + C = 0$ is $\frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$.

Section 1.4

2. If $f(x) = ab^x$, then $\frac{f(x+h)}{f(x)} = \frac{ab^{x+h}}{ab^x} = b^h$, independent of x . If $g(x) = mx + b$, then $g(x+h) - g(x) = [m(x+h) + b] - (mx + b) = mh$, independent of x .

4. (b) The function in (a) gives an 1860 estimate of 33.38 million and an 1870 estimate of 45.36

million. The increased discrepancy for 1870 is probably due to the Civil War.

6. $f(x) = 2.25(\frac{3.38}{2.25})^x; g(t) = -6.4t - 1.3; s(u) = 1.5(0.95)^u$.

7. (b) $y = 2 \cdot 3^{x/3}$. (d) $y = 1 + \frac{3}{2}(\frac{2}{3})^{x/2}$.

Section 1.5

1. (b) sidereal (earth-moon only): 27 d 7 h 43 m; synodic (phases): 29 d 12 h 44 m. (d) one hour.
(f) $\frac{9}{10\pi}$ sec. (h) $\frac{1}{1500}$ sec.

2. (c) $\cos(a+b) = \cos[a - (-b)] = \cos a \cos(-b) + \sin a \sin(-b) = \cos a \cos b - \sin a \sin b$. (e)
 $\sin(a-b) = \cos[\frac{\pi}{2} - (a-b)] = \cos[(\frac{\pi}{2} - a) + b] = \cos(\frac{\pi}{2} - a) \cos b - \sin(\frac{\pi}{2} - a) \sin b = \sin a \cos b - \cos a \sin b$.

(g) $\tan(x-y) = \frac{\sin(x-y)}{\cos(x-y)} = \frac{\sin x \cos y - \cos x \sin y}{\cos x \cos y + \sin x \sin y} = \frac{\tan x - \tan y}{1 + \tan x \tan y}$. (i) $\tan^2 t = \frac{\sin^2 t}{\cos^2 t} = \frac{1 - \cos 2t}{1 + \cos 2t}$ (you
could stop here, or) $= \frac{(1 - \cos 2t)^2}{1 - \cos^2 2t} = \frac{(1 - \cos 2t)^2}{\sin^2 2t} \Rightarrow \tan t = \frac{1 - \cos 2t}{\sin 2t}$ (and you could stop here,
or) $= \frac{1 - \cos^2 2t}{\sin 2t(1 + \cos 2t)} = \frac{\sin 2t}{1 + \cos 2t}$.

5. The sides of a "30-60-right" triangle are in the ratio 1, $\sqrt{3}$, 2, giving the ratios in the table.

9. (b) $y = 4 \sin x$. (d) $y = 1 - \cos 2x$. (f) $y = -24 \cos \frac{\pi}{9}$. (h) $y = 1 - \sin \pi x$.

18. If the line $y = c$ meets the sine curve at x_1 , then it also meets it at $\pi - x_1$, from which it follows that all the solutions of $\sin x = c$ are $x_1 + 2n\pi$ and $\pi - x_1 + 2n\pi, n \in \mathbb{Z}$.

21. (a) about June 3; 4110. (b) about Jan 26; 2890. (c) Apr 1; 3500. (d) about Aug 1, 3750; and about Dec 1, 3230.

Section 1.6

1. (b) $2-x-x^2; -x-x^2; (1-x)(x^2+1); \frac{1-x}{x^2+1}; (1-x)^2+1; -x^2$. (d) $e^x + \sin x; e^x - \sin x; e^x \sin x; \frac{e^x}{\sin x}; \sin(e^x); e^{\sin x}$. (g) $1 - \sin x + \sqrt{x}; 1 - \sin x - \sqrt{x}; (1 - \sin x)\sqrt{x}; \frac{1 - \sin x}{\sqrt{x}}; \sqrt{1 - \sin x}; 1 - \sin \sqrt{x}$. (h)
 $\frac{x^2 + 3x - 1}{(x+2)(x-1)}; \frac{-x^2 - x - 1}{(x+2)(x-1)}; \frac{x}{(x+2)(x-1)}; \frac{x-1}{x(x+2)}; \frac{-1}{x+1}; \frac{x-1}{3x-2}$.

2. (b) $\frac{1}{3x+2}$. (d) $\frac{15x+7}{3x+2}$.

5. 25 dollars.

6. (a) $x^3 + 2x^2 + 10x + 30 + \frac{87}{x-3}$. (c) $3x^3 - 3x^2 + 2x - 5 + \frac{7x+1}{x^2+x-1}$.

7. (a) 1 is a zero of $x^b - 1$, so $x - 1$ is a factor.

8. (a) Division Algorithm: Given polynomials $f(x)$ and $g(x)$, there exist polynomials $q(x)$ and $r(x)$ such that $f(x) = g(x)q(x) + r(x)$ and either $r(x) = 0$ or the degree of r is less than the degree of g .

(b) Remainder Theorem: If $p(x)$ is a polynomial and a is a real number, then there is a polynomial $q(x)$ such that $p(x) = q(x)(x - a) + p(a)$. (c) Factor Theorem: $x - a$ is a factor of polynomial $p(x)$ if and only if $p(a) = 0$.

9. $4, 1 \pm \sqrt{3}$.

10. $(x + 1)(x - 1)(x - 3)(x - \frac{3}{4})$.

11. $p = b - \frac{1}{3}a^2; q = \frac{2}{27}a^3 - \frac{1}{3}ab + c$.

13. $y = \frac{16}{25}(x - \frac{7}{2})^2$.

15. (b) $[-3, 7] \times [-2, 8]$.

20. $[0, 4] \times [0, 4]; [0, 140] \times [0, 1000000]; [0, 210] \times [0, 8000000]$.

Section 1.7

1. (b) $2x - 5; \frac{x+5}{2}$. (d) $(x + 1)^3; x^{1/3} - 1$. (f) $\frac{x+1}{2x}; \frac{1}{2x-1}$. (h) $(x + 2)^4$; no inverse function.

2. (b) Yes, $(x + 4)^5$. (d) No. (f) Yes, $\frac{x}{3x-1}$.

5. in order from top down.

9. If the graph of the function is a parabola with vertex (a, b) , restrict the domain to either $x \geq a$ or $x \leq a$, taking the appropriate square root in solving.

10. If the horizontal line $y = c$ meets the graph of f at points (x_1, c) and (x_2, c) , then $f(x_1) = f(x_2) = c$ but $x_1 \neq x_2$, so f cannot be one-to-one.

Section 1.8

1. (b) $\log_3 7 - 4$. (d) $2x + 3 + \ln 4$. (f) $\ln(1 + x) - 3x$.

3. (b) $\frac{-\ln 5}{\ln 3 - 2 \ln 5}$. (d) No solution. (f) $3e^{-2}$.

4. $t = D \Rightarrow Q = 2Q_0 \Rightarrow 2Q_0 = Q_0 e^{kD} \Rightarrow e^{kD} = 2 \Rightarrow kD = \ln 2$.

7. $k = \frac{\ln 2}{h} \Rightarrow Q = Q_0 e^{-kt} = Q_0 e^{-(t/h) \ln 2} = Q_0 (e^{\ln 2})^{-t/h} = Q_0 2^{-t/h}$.

11. \$182.21

14. (b) 72.9%.

19. 6.1837%.

23. 254.6 million years.

29. (b) 2.303. (d) 0.683. (e) 6.406

Section 1.9

1. (b) $\frac{\pi}{6}$. (d) $\frac{5\pi}{6}$. (f) 0.85707. (h) 1. (j) $\frac{1}{2}(e - e^{-1})$.

2. Let $\theta = \sin^{-1} x$. Then $x = \sin \theta = \cos(\frac{\pi}{2} - \theta) \Rightarrow \cos^{-1} x = \frac{\pi}{2} - \theta = \frac{\pi}{2} - \sin^{-1} x$, from which the identity follows. Alternatively, $\sin^{-1} x$ and $\cos^{-1} x$ are acute angles of a right triangle, so add up to $\frac{\pi}{2}$.

6. (c) $-\frac{\pi}{6} + 2n\pi, -\frac{5\pi}{6} + 2n\pi$. (e) $\frac{\pi}{2} + n\pi, \frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi$.

9. (b) $-\frac{5\pi}{6}$.

11. (b) $\frac{2\sqrt{2}}{3}, \frac{1}{3}, 2\sqrt{2}$. (d) Correction: $x = \sec^{-1} \frac{3}{2}$. $\frac{\sqrt{5}}{3}, \frac{2}{3}, \frac{\sqrt{5}}{2}$.

12. (a) If $x > 0$, let $\cot^{-1} x = \theta$. Then $\cot \theta = x = \frac{1}{\tan \theta} \Rightarrow \tan \theta = \frac{1}{x} \Rightarrow \theta = \tan^{-1} \frac{1}{x}$. If $x < 0$, we add π to $\tan^{-1} \frac{1}{x}$ to put $\cot^{-1}$ into Quadrant II instead of Quadrant IV. (b) As in part (a), for $x > 0$, use the fact that $\sec \theta = \frac{1}{\cos \theta}$. For $x < 0$, changing the sign puts us in Quadrant III where we belong. (c) Similar to (b).

14. Let $\theta = \tan^{-1} \frac{1}{5}$. Then $\tan 2\theta = \frac{5}{12}, \tan 4\theta = \frac{120}{119}$. If $\phi = \tan^{-1} \frac{1}{239}$, then $\tan(4\theta - \phi) = \frac{120 \cdot 239 - 119}{119 \cdot 239 + 120} = 1$. The result follows.

15. (a) $\cosh^2 x - \sinh^2 x = (\frac{e^x + e^{-x}}{2})^2 - (\frac{e^x - e^{-x}}{2})^2 = \frac{e^{2x} + 2 + e^{-2x}}{4} - \frac{e^{2x} - 2 + e^{-2x}}{4} = \frac{4}{4} = 1$.