Solutions to Some Problems, Calculus (Third Ed.)

Chapter 2

Section 2.1

1. (b) 16. (d) –16.

2. (b) 19.8.

3. 9; 7; 6; 5.2; 5.1; about 5.

5. (b) 40.6; 5.4; 0.7. (c) approaches 150; approaches 0.

9. about –2; no tangent at –1 or 2.

14. $b \to a$ if and only if $h \to 0$; substitute.

17. (a) $\frac{C(b)-C(a)}{b-a}$. (b) $\lim_{b\to a} \frac{C(b)-C(a)}{b-a}$.

22. (a) $3q + 0.005q^2 - 0.001q^3 - 4000$. (b) 0.426. (c) 0.514. (d) 0.59149. (e) about 0.6.

Section 2.2

1. (b) –1. (d) 0. (f) $\frac{1}{2}$. (h) DNE.

2. (b) 6. (d) –1. (f) –5. (h) DNE. (j) DNE. (l) 0. (n) 0. (p) DNE.

3. (b) $\delta \le 0.003$.

4. (b) $\delta \le 0.003$.

5. (b) $\delta \le 0.0024$.

6. (b) Given $\epsilon > 0$, let $\delta \le 3\epsilon$. Then $0 < |x-9| < \delta \Rightarrow |\sqrt{x} - 3| = \frac{1}{|\sqrt{x}+3|}|x-9| < \frac{1}{3}\delta \le \epsilon$. (d) Given $\epsilon > 0$, let $\delta \le \frac{1}{2}\epsilon$. $|x+1| < \delta \Rightarrow -1-\delta < x < -1+\delta \Rightarrow |x| > 1+\delta \Rightarrow \frac{1}{|x|} < 1$. Hence $0 < |x+1| < \delta \Rightarrow |\frac{2}{x} + 2| = 2\frac{1}{|x|}|x+1| < 2\delta \le \epsilon$.

13. (a) close to 12.

14. (d) 1. (e) 1. (f) 1. (j) 0. (k) 0. (l) 0.

Section 2.3

1. (b) 1. (d) DNE. (f) $\frac{3}{4}$. (h) DNE. (j) –4. (l) –6.

2. (b) $-\frac{1}{2}$. (d) DNE. (f) $\frac{2}{3}$. (h) $\frac{1}{2\sqrt{2}}$. (j) $-\frac{1}{4}$. (m) $\frac{3}{4}$.

4. For example, $f(x) = \frac{|x|}{x}$ at 0.

5. Copy the proof of Thm. 17, using $|m - g(x)| = |g(x) - M|$.

Section 2.4

1. (b) $\frac{1}{\sqrt{3}}$. (d) –2. (f) 2. (h) $\frac{2}{3}$. (j) 4. (l) DNE.

2. (b) 0. (d) 0. (f) 1. (h) DNE. (j) DNE.

3. (b) e^{-2}. (d) DNE. (f) 10^{-3}.

5. Suppose $L > M$. Let ϵ be small enough that $M + \epsilon < L - \epsilon$. Then we can choose δ such that $0 < |x - c| < \delta \Rightarrow |f(x) - L| < \epsilon$ and $|g(x) - M| < \epsilon$. But then $g(x) < M + \epsilon < L - \epsilon < f(x)$, contradicting $f(x) \leq g(x)$. Hence $L \leq M$.

7. (a) 0. (b) 0. (c) 0. (d) For example, let $f(x) = \frac{1}{a} \sin ax$ and $g(x) = \sin^2 ax$ for two values of a. (e) can't tell.

8. (b) 0, 0, 0. (d) 0, –1, DNE.

10. (b) 1. (d) between –1 and 0.

11. (b) No. (d) No. (f) $B = 1$.

12. Yes. the limit is 0.

13. (b) $P_n = 2rn \sin \frac{\pi}{n}$. (d) $A_n = nr^2 \sin \frac{\pi}{n} \cos \frac{\pi}{n} = \pi r^2 \frac{\sin(\pi/n)}{\pi/n} \cos \frac{\pi}{n} \to \pi r^2$.

Section 2.5

1. (b) $\frac{1}{2}$. (d) 0. (f) $\frac{\pi}{2}$. (h) 2. (j) $-\frac{1}{2}$. (l) 1. (n) 0.

5. (c) 0. (d) $-\frac{\pi}{2}$. (f) 1. (h) 1.

8. (b) $y = 1$. (d) $y = \pm\frac{\pi}{2}$. (f) $y = 0$.

9. (b) ∞. (d) $-\infty$. (f) ∞. (h) $-\infty$. (j) ∞

10. (a) $f \to 2$ as $x \to \pm\infty$; $g \to 0$ as $x \to \pm\infty$.

11. (b) $f \to \infty$ as $x \to \pm\infty$. (d) $f \to \infty$ as $x \to \pm\infty$. (f) $f \to 0$ as $x \to \pm\infty$. (h) $f \to 0$ as $x \to \pm\infty$. (j) $f \to \infty$ as $x \to \infty$; $f \to -\infty$ as $x \to -\infty$.

12. (e) $x = 0$. (f) $x = 1; x = -1$. (g) none. (h) none.

13. (e) $y = 0; x = 1, x = -1$. (f) none; $x = 0$. (h) $y = 1; x = 0$.

Section 2.6

1. (b) Let $f(3) = 6$. (d) Let $f(\frac{\pi}{2}) = 0$. (f) Let $f(2) = \frac{1}{2}$.

2. (b) $\lim_{x\to n-} \lfloor x \rfloor = n - 1; \lim_{x\to n+} \lfloor x \rfloor = n$. (d) $f \to 0$ as $x \to \pi-$; $f \to -1$ as $x \to \pi+$.

3. (b) $\lim_{x\to 3+} \frac{1}{x-3} = \infty$. (d) $f \to -\infty$ as $x \to 0+$.

4. (d) infinite discontinuity at $\frac{\pi}{2}$. (e) jump discontinuity at $\frac{\pi}{2}$. (f) removable discontinuity at 0.

5. (b) $x \neq -1$. (d) on $\mathbb{R}$. (f) on $(-2, 2)$.

7. x in the domain requires $x < c$, so $x \to c-$.

11. The discontinuity is essential because the limit of $\sin\frac{1}{x}$ as $x \to 0$ does not exist. It is not a jump discontinuity because neither one-sided limit exists; it is not infinite because neither one-sided limit is infinite.

12. $g(f(x)) = \frac{1}{x}$. g is not continuous at $f(0)$.

13. The limit at 0 DNE, and neither do the one-sided limits; neither is a one-sided limit ∞ or $-\infty$.

Section 2.7

1. (b) No. (d) No. (f) Yes. (h) Yes.

2. (a) the interval is open. (b) the function is discontinuous (the discontinuities are removable).

3. (b) Let $\epsilon = \frac{1}{2}|f(c)|$. Then there is $\delta > 0$ such that $|x - c| < \delta \Rightarrow |f(x) - f(c)| < \epsilon \Rightarrow f(x) < \frac{1}{2}f(c) < 0$, so f is negative throughout $(c - \delta, c + \delta)$.

5. Let F be the amount of fuel left at time t and let D be the distance traveled by time t. Then $F - D$ starts positive and ends negative, so must be zero at some point T, assuming continuity. Hence $F(T) = D(T)$.

6. (b) Since f is continuous near c, we have $\lim_{x\to c} f(x) = f(c)$, so that for any $\epsilon > 0$ there exists $\delta > 0$ such that if $c-\delta < x < c+\delta$, then $|f(x)-f(c)| < \epsilon$, which means $f(c)-\epsilon < f(x) < f(c)+\epsilon$.

7. (b) g is continuous on $[a, b]$, hence bounded by 6(d). (d) If x is such that $f(x) > M - \epsilon$, then $M - f(x) < \epsilon$ and $g(x) = \frac{1}{M-f(x)} > \frac{1}{\epsilon} > B$.

11. Let m, M be points at which f has its minimum and maximum. Then by Problem 7, there is c between m and M such $f(c) = v$, so the IV Theorem is true.

13. f is continuous at every point of (c, d). The two-sided limits of f at c and at d exist, so the appropriate one-sided limits do, too.

14. (a) Let x_1, x_2 be chosen so that $a < x_1 < x_2 < b$. Because f is one-to-one, we know $f(x_1) \neq f(x_2)$. Suppose $f(x_1) < f(x_2)$ (the other case is similar). Now we show that f is increasing in (x_1, x_2); if not, there exist points x_3, x_4 such that $x_1 < x_3 < x_4 < x_2$ and $f(x_3) > f(x_4)$. We now have three cases: (i) If $f(x_3) < f(x_1)$, then a horizontal line will meet the graph of f between x_1 and x_3 and again between x_3 and x_2. (ii) If $f(x_4) > f(x_2)$, then a horizontal line will meet the graph of f between x_1 and x_4 and again between x_4 and x_2. (iii) If $f(x_3) > f(x_1)$ and $f(x_4) < f(x_2)$, then a horizontal line will meet the graph of f between x_1 and x_3, between x_3 and x_4, and between x_4 and x_2. In all three cases, we contradict the fact that f is one-to-one. Finally, we show that f is increasing in (a, b). If not, then either f decreases somewhere between a and x_1 or somewhere between x_2 and b; since these cases are similar, we will demonstrate only the first. Suppose there are points x_5, x_6 such that $a < x_5 < x_6 < x_1$ and $f(x_5) > f(x_6)$. If $f(x_6) < f(x_1)$, then a horizontal line will meet the graph of f between x_5 and x_6 and again between x_6 and x_1. If $f(x_6) > f(x_1)$, then a horizontal line will meet the graph of f between x_6 and x_1 and again between x_1 and x_2. In both cases, we contradict the fact that f is one-to-one.

(b) Let y_1, y_2 be in the domain of f^{-1}, and suppose $y_1 < y_2$. Let $f(x_i) = y_i$. Then $f(x_1) < f(x_2) \Rightarrow x_1 < x_2$. For if $x_1 > x_2$, then f is increasing and has to preserve order, contradicting the order of y_1 and y_2. Hence $f^{-1}(y_1) = f^{-1}(f(x_1)) = x_1 < x_2 = f^{-1}(f(x_2)) = f^{-1}(y_2)$, and f^{-1} is increasing.

(c) If f is increasing, then f^{-1} is also increasing. Hence $f(d-\epsilon) < x < f(d+\epsilon) \Rightarrow d-\epsilon < f^{-1}(x) < d+\epsilon \Rightarrow |f^{-1}(x) - d| < \epsilon$. If f is decreasing, we define δ by $f(d+\epsilon) < c-\delta < x < c+\delta < f(d-\epsilon)$, and use the fact that f^{-1} is also decreasing. Hence $f(d+\epsilon) < x < f(d-\epsilon) \Rightarrow d+\epsilon > f^{-1}(x) > d-\epsilon$, as before.

16. (b) $p(-2) = 12, p(-1) = -1$. (d) $p(0) = 4, p(1) = 2$; can't tell.

17. (b) [0.6875,0.75].

18. $\dfrac{L}{2^n}$.