Solutions to Some Problems, Garner's *Calculus* (Third Ed.)

Chapter 3

Section 3.1

1. (b) –2. (d) $y = -2x + 3$.

2. (b) The graph straightens out and appears to have a slope of about 0.9. (d) The graph appears to flatten out, indicating that probably $f'(0) = 0$.

3. (b) 6. (d) $\frac{1}{4}\sqrt{15}$.

4. (b) $2ax + b$. (d) $\frac{-1}{2x\sqrt{x}}$.

5. (b) No; no.

6. (b) Since $\sin\frac{1}{x}$ is bounded, $x\sin\frac{1}{x} \to 0$ as $x \to 0$. But $g'(0) = \lim_{h\to 0}\frac{h\sin(1/h)-0}{h} = \lim_{h\to 0}\sin\frac{1}{h}$ does not exist.

7. (b) $-32t; -32$. (d) $-\frac{1}{t^2}; \frac{2}{t^3}$.

9. (b) 2. (d) 24.

10. 2π.

13. $250 - 0.8q; 10$.

16. The slope is 0.

19. (a) They are estimates to the slope.

(b) $$\frac{1}{2}\left[\frac{f(x+h)-f(x)}{h} + \frac{f(x)-f(x-h)}{h}\right] = \frac{f(x+h)-f(x-h)}{2h}.$$

(c) It is an estimate that is the average of the previous two. (d) Central is likely closer.

Section 3.2

1. (b) $8x^3$. (d) $15x^{-4}$. (f) 0. (h) $15x^4-5$. (j) $3x^2+3x^{-4}$. (l) $16x^3+9x^2$.

2. (b) $(10x^4-8x)(3x^{10}+x^2)+(2x^5-4x^2)(30x^9+2x)$. (d) $6(2x+1)^2$. (f) $-(4x^3+2x)(x^4+x^2-1)^{-2}$. (h) $\dfrac{2x^2-6x-2}{(2x-3)^2}$. (j) $\dfrac{2x^4+16x^3+x^2+2x+4}{(x^2+4x)^2}$. (l) $\dfrac{a^2-x^2}{(x^2+a^2)^2}$.

(n) $(x-b)(x-c)+(x-a)(x-c)+(x-a)(x-b)$.

3. (b) $-32t; -32; 144; -128; -32$. (d) $2t-2t^{-3}; 2+6t^{-4}; \frac{626}{25}; \frac{1248}{125}; \frac{1256}{625}$.

4. (b) $60x^3$. (d) $56x^{-9}$. (f) $\dfrac{-2abc}{(bx+c)^3}$.

5. (b) $\dfrac{(-1)^n n! 2^n}{(2x+a)^{n+1}}$. (d) $\begin{cases} k(k-1)\cdots(k-n+1)x^{k-n}, & n \le k \\ 0, & n > k \end{cases}$

6. (b) $y=2x+3$. (d) $y=4$.

7. (b) $y=-\frac{1}{2}x+\frac{1}{2}$. (d) $x=2$.

8. (b) $4\pi r^2$. (d) $6e^2$.

18. $(fghj)' = (fgh)'j + (fgh)j' = [f'gh+fg'h+fgh']j + fghj' = f'ghj + fg'hj + fgh'j + fghj'$.

Section 3.3

3. (b) $(2x+x^2)e^x$. (d) $2e^{2x}$. (f) $(2\sin x+\cos x)e^{2x}$. (h) $2(\cos^2 x-\sin^2 x) = 2\cos 2x$. (j) $2\sin x\cos x$. (l) $\dfrac{-1}{x^2}+\dfrac{\cos x}{\sin^2 x}$

4. (b) $\dfrac{2x\cos x+(x^2-1)\sin x}{\cos^2 x}$. (d) $\dfrac{(x+5)\cos x+(x+3)\sin x}{(x+4)^2}$. (f) $(1+x\ln 2)2^x$. (h) $3e^{3x}$. (j) $4e^{4x}(\sin x+\cos x)+e^{4x}(\cos x-\sin x)$. (l) $-2(\ln 3)3^{-2x}$.

5. (b) $(x+2)e^x$. (d) $2e^x\cos x$. (f) $2\sec^2 x\tan x$.

6. $\cos x, -\sin x, -\cos x, \sin x, \cos x, -\sin x; \cos x$.

8. $\frac{d}{dx}(\cos x) = \lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h} = \lim_{h\to 0}\frac{\cos x\cos h-\sin x\sin h-\cos x}{h} = \lim_{h\to 0}[\cos x\frac{\cos h-1}{h} - \sin x\frac{\sin h}{h}] = -\sin x$.

10. $\frac{d}{dx}(\cot x) = \frac{d}{dx}(\frac{\cos x}{\sin x}) = \frac{-\sin x\sin x-\cos x\cos x}{\sin^2 x} = \frac{-1}{\sin^2 x} = -\csc^2 x$.

13. (b) $-3\sin t; -3\cos t; 0; 3$. (d) $1+5\sin t; 5\cos t; 6; 0$.

14. (b) $t = 2\pi$.

15. (b) $y = 1$. (d) $y = -\frac{\sqrt{2}}{2}x + \frac{\pi}{4} + \frac{\sqrt{2}}{2}$. (f) $y = -\frac{1}{e}x + \frac{2}{e}$.

16. For example, at $h = 0.001$, $\frac{e^h-1}{h} = 1.00050017$. $\dfrac{d}{dx}(e^x) = \lim_{h\to 0} \dfrac{e^{x+h}-e^x}{h} = e^x \lim_{h\to 0} \dfrac{e^h - 1}{h} = e^x \cdot 1 = e^x$.

Section 3.4

1. $\frac{dI}{dD}$ is the rate of change of intensity w.r.t. depth, in lumens per foot. $\frac{dD}{dL}$ is the rate of change of depth w.r.t. distance, in feet per rod. $\frac{dI}{dL}$ is the rate of change of intensity w.r.t. distance, in lumens per rod.

3. (b) $5(2x-x^2)(2-2x)$. (d) $3(1-\frac{1}{x})^2(1+\frac{1}{x^2})$. (f) $3(1-\cos x)^2\sin x$. (h) $5\sec^2 5x$. (j) $2xe^{x^2}$. (l) $2^{\sin x}(\ln 2)\cos x$.

4. (b) $3\sec 3x\tan 3x\tan 4x + 4\sec 3x\sec^2 4x$. (d) $-5\pi\cos^4 \pi x\sin \pi x$. (f) $-e^{-x}(\cos 3x + 3\sin 3x)$. (h) $-\sec^2 xe^{\tan x}\sin(e^{\tan x})$. (j) $\dfrac{-12\cos 3x\sin 3x}{1-\cos 3x} - \dfrac{3(\cos^4 3x - \sin^4 3x)\sin x}{(1-\cos 3x)^2}$. (l) $\dfrac{7e^{-0.02x}}{(1+7e^{-0.02x})^2}$.

5. (b) $8u$. (d) $6u\cos 3x$. (f) $9u^2x^2$.

6. (b) (iii). (d) (x).

9. –28.

11. $-6x(1-x^3)$.

13. (b) $-\frac{1}{3}$.

18. f is odd $\Rightarrow f(-x) = -f(x) \Rightarrow f'(-x)(-1) = -f'(x) \Rightarrow f'(-x) = f'(x) \Rightarrow f'$ is even.

19. (a) $\frac{\pi}{180}\cos x^\circ$. (b) the same. (c) We avoid the extra factor of $\frac{\pi}{180}$.

Section 3.5

1. (b) $\dfrac{x}{2y}$. (d) $\dfrac{6x^5-2xy^3}{3x^2y^2+5y^4}$. (f) $\dfrac{y-y^4}{3x+y^4}$. (h) $-\dfrac{y^{1/3}}{x^{1/3}}$. (j) $\dfrac{\cos x}{3y^2}$. (l) $-\dfrac{y|x|}{x|y|}$.

2. (b) $\dfrac{x}{2y}; \dfrac{2y^2-x^2}{4y^3} = \dfrac{-3}{4y^3}$. (d) $-x^{-1/3}y^{1/3}; \frac{1}{3}x^{-4/3}y^{-1/3}(x^{2/3}+y^{2/3}) = \frac{1}{3}x^{-4/3}y^{-1/3}$.

(f) $\dfrac{x-y}{x+y}$; $\dfrac{-2(x^2-2xy-y^2)}{(x+y)^3}=0$.

3. (b) $3x|x|$. (d) $\dfrac{2x(x^2-4)}{|4-x^2|}$. (f) $\dfrac{2x}{x^2+4}$. (h) $\cot x$. (j) $\sec x$. (l) $\dfrac{1}{x[1+(\ln x)^2]}$. (n) $\dfrac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}+\dfrac{e^x}{\sqrt{1-e^{2x}}}$. (p) $\dfrac{-\cos(\tan^{-1}x)\sin(\tan^{-1}x)}{\sqrt{1+\cos^2(\tan^{-1}x)}}$

4. (b) $\dfrac{1}{2\sqrt{x+3}}$. (d) $\frac{1}{3}x^{-2/3}-x^{-4/3}$. (f) $-\frac{4}{5}x^{-7/5}+\frac{2}{15}x^{-5/3}$. (h) $\frac{1}{3}\sin^{-2/3}x\cos x$. (j) $\frac{2}{3}x^{-1/3}\sec(x^{2/3})\tan(x^{2/3})$. (l) $\frac{1}{2\sqrt{x}}e^{\sqrt{x}}-\frac{1}{2\sqrt{x}}e^{-\sqrt{x}}$.

5. (b) $y=\frac{1}{\sqrt{2}}+\sqrt{2}(x-2)$. (d) $y=\dfrac{3\sqrt{3}}{8}-3(x+\dfrac{1}{8})$.

7. $\dfrac{-1}{1+x^2}$.

8. If the principal domain is in quadrants I and III, then $y=\sec^{-1}x\Rightarrow x=\sec y\Rightarrow 1=\sec y\tan y\dfrac{dy}{dx}\Rightarrow\dfrac{dy}{dx}=\dfrac{1}{\sec y\tan y}=\dfrac{1}{x\sqrt{x^2-1}}$. The slope is positive if x is positive, negative if x is negative. If the principal domain is in quadrants I and II, the slope is always positive, so $\dfrac{dy}{dx}=\dfrac{1}{|x|\sqrt{x^2-1}}$.

9. (b) By the chain rule, $\dfrac{d}{dx}[\ln f(x)]=\frac{1}{f(x)}f'(x)$.

10. (b) $[\frac{1}{x}\ln x-\ln x\tan x](\cos x)^{\ln x}$. (d) $\left[\dfrac{7}{2(4-x)}-\dfrac{3}{2(1-x)}\right]\sqrt{\dfrac{(1-x)^3}{(4-x)^7}}$.