Solutions to Some Problems, *Calculus* (Third Ed.)

Chapter 4

Section 4.1

2. 10.981; 10.8.

4. 14.641π; 14.4π.

6. 6%.

8. increases by 3%.

10. (b) 1.01004512021; 1.01. (d) 0.8885; 0.88. (f) 3.97905720789; 3.979166.... (h) -0.01005; -0.01. (j) –0.00999983; –0.01. (l) 1.030454; 1.03. (n) 1.10517091808; 1.1.

11. (b) $(2x+2)dx$. (d) $4\cos 4x\ dx$. (f) $6\sin 3x\cos 3x\ dx$. (h) $e^{\sin^{-1}x}(1-x^2)^{-1/2}\ dx$.

12. 0.1232π.

Section 4.2

1. (b) 0. (d) $-2^{-1/3}$. (f) 0. (h) –2. (j) $\frac{\pi}{6}+2n\pi, \frac{5\pi}{6}+2n\pi, \frac{3\pi}{2}+2n\pi$. (l) $\frac{2\pi}{3}+2n\pi, \frac{4\pi}{3}+2n\pi, n\in\mathbb{Z}$. (n) none. (p) $\frac{\pi}{2}+n\pi, n\in\mathbb{Z}$.

2. (b) increasing and concave downward. (d) increasing and concave downward.

3. Since f is a polynomial of odd degree, it has at least one real zero. If it has two, then between them is a critical point, by Rolle's Theorem. But $f'(x)=-(1+18x^2)<0$, so there is no critical point. Hence f has exactly one real zero.

6. By Problem 9, there are two times at which the speeds are the same, or the difference in speeds is zero. Hence between them is a time at which the derivative of the difference in speeds is zero, by Rolle's Theorem; at that time, the accelerations are the same.

7. (b) $\cos^{-1}(-\frac{2}{3\pi})\approx 1.7846$. (d) $e^{\ln(1+e)/e}-1\approx 0.6211$.

8. (b) There is a discontinuity at 0. (d) f is not differentiable at 0.

10. By the Mean Value Theorem applied to f over $[a, x]$, there is a point c such that $f(x) - f(a) = f'(c)(x - a)$. Hence $|f(x) - f(a)| = |f'(c)||b - a| \leq M|x - a|$.

Section 4.3

1. (b) probably increasing and concave downward. (d) increasing and concave downward.

5. (b) critical point: $\frac{3}{2}$; decreasing in $(-\infty, \frac{3}{2})$; increasing in $(\frac{3}{2}, \infty)$; no hypercritical points; concave up on $(-\infty, \infty)$. (f) no critical points; increasing in $(-\infty, -1) \cup (-1, \infty)$; no hypercritical points; concave up in $(-\infty, -1)$; concave down in $(-1, \infty)$. (j) critical points: $\frac{n\pi}{2}$; increasing in $(n\pi, \frac{\pi}{2} + n\pi)$; decreasing in $(\frac{\pi}{2} + n\pi, \pi + n\pi)$; hypercritical points: $\frac{\pi}{4} + \frac{n\pi}{2}$; concave upward in $(-\frac{\pi}{4} + \frac{n\pi}{2}, \frac{\pi}{4} + \frac{n\pi}{2})$; concave downward in $(\frac{\pi}{4} + \frac{n\pi}{2}, \frac{3\pi}{4} + \frac{n\pi}{2})$; all hypercritical points are inflection points. (n) critical point: 1; increasing in $(-\infty, 1)$; decreasing in $(1, \infty)$; hypercritical point: 2; concave down in $(-\infty, 2)$; concave up in $(2, \infty)$; inflection point: 2.

6. (b) $y = 7 - 15(x - 1); y = 0; y = -7 - 15(x + 1)$. (d) $y = \pi - x$.

7. (a) $y = e^a + e^a(x - a)$. (b) $e^a(1 - a)$. (c) Because the curve's intercept is 1 and every tangent line is below the curve (the curve is concave upward), the intercept will never be greater than 1.

8. Because $p - c, d - c$, and $d - p$ are all positive, both inequalities reduce by cross-multiplying to the inequality $df(p) - df(c) - cf(p) \leq pf(d) - pf(c) - cf(d)$.

17. (b) vertical tangent line. (d) vertical tangent line. (f) corner. (h) vertical tangent line. (j) vertical tangent line.

19. (b) between 60 and 72 degrees. (d) between 32.5 and 38.75 minutes.

Section 4.4

1. (b) global minimum at $(-1, 2)$. (d) local maximum at $(-\frac{1}{2}, \frac{45}{4})$; local minimum at $(4, -171)$. (f) global maximum at $(2^{-1/3}, \frac{1}{4})$. (h) global minimum at $(\frac{5}{2}, -\frac{1}{4})$. (j) global maximum at $(3, \frac{8}{3})$. (l) none. (n) global minimum at (0, 0). (p) global minima at $(\pm 2, 0)$; global maximum at (0, 2).

2. (b) global minimum at $(-1, -1)$; local minimum at (1, 1); global maximum at $(\frac{1}{\sqrt{2}}, \sqrt{2})$. (d) global minima at $(\sin^{-1}(-\frac{1}{4}) + 2n\pi, -\frac{9}{8})$ and $(\pi - \sin^{-1}(-\frac{1}{4}) + 2n\pi, -\frac{9}{8})$; local maxima at $(\frac{3\pi}{2} + 2n\pi, 0)$; global maxima at $(\frac{\pi}{2} + 2n\pi, 2)$. (f) local minima at $(\frac{\pi}{3} + 2n\pi, \frac{\pi}{3} + 2n\pi - \sqrt{3})$; local maxima at $(\frac{5\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi + \sqrt{3})$. (h) global minima at $(\frac{\pi}{2} + n\pi, 0)$; global maxima at $(n\pi, \frac{1}{2})$. (j) global minimum at $(\frac{1}{3}, -\frac{2}{3e})$. (l) global maximum at (0, 3). (n) none. (p) none.

4. If f has an inflection point at c, then f' changes sign at c, so f' has an extremum at c.

5. (b) $(x+2)e^x$. (d) $2e^x \cos x$.

6. Yes; if the maximum is not global, then it is not the only maximum.

8. (a) Choose $y = \frac{27-x}{4}$. (b) Choose $x = -1$. (c) Choose $x = \frac{y-9}{2}$. (d) Choose $y = 7$. (e) Choose $x = -1, y = 7$.

Section 4.5

2. Cut out a square of side $\frac{1}{6}s$ from each corner.

4. 25 feet by 10 feet.

6. $W = \frac{2r}{\sqrt{3}}; D = \frac{2r\sqrt{2}}{\sqrt{3}}$.

8 (b) $s = r\sqrt{3}; A = \frac{3\sqrt{3}}{4}r^2$.

10. $\frac{32}{3\sqrt{3}}$.

12. $\frac{3\sqrt{3}}{4}L^2$.

18. $h = 2(\frac{v}{2\pi})^{1/3} = 2r$.

20. \$1020.

21. $r'(P) = f'(P) - 1 = 0 \Rightarrow f'(P) = 1$.

22. $r(5) = 25$. That is, the maximum sustainable removal occurs when the population is 5000 and 25,000 per year can be removed.

24. $2\sqrt{2}$ feet.

26. $(\frac{1}{2}, \sqrt{\frac{7}{2}})$.

28. At a point $\dfrac{6^{1/3}}{1+6^{1/3}} \approx 0.645$ of a block from the brighter light.

30. almost 9:00 am.

32. (a) between about 50.3° north latitude and 50.3° south latitude. (b) between about 66° north latitude and 66° south latitude. (c) at about 34° north latitude or 34° south latitude; yes.

34. (a) at $\frac{8}{3}$ mile from the point on the river nearest the closer ranch. (b) to the closer ranch and

then to the farther one.

35. (a) hovering takes more energy than flight, because of lift. (b) near the first tic mark. (c) $a(v) = \frac{f(v)t}{vt} = \frac{f(v)}{v}$. (e) near the second tic mark. (f) minimize f if the aim is to stay aloft; minimize a if the aim is to get somewhere.

Section 4.6

2. (a) about \$50. (b) about 3.5 tons per day. (c) about 4 tons per day. (d) about 6 tons per day. (e) about 7 tons per day. (f) about \$250 per day.

4. 100 sets; 250 sets.

6. (a) $C = 2400 - 40p; R = 280p - 8p^2$. (b) $P = 320p - 8p^2 - 2400$. (c) $a = \frac{300-5p}{35-p}$. (d) \$20; 120. (e) \$17.50; 140.

8. 30.

10. (a) $C = 49 + 0.18q; a = 0.18 + \frac{49}{q}$. (b) $q = 480 - 80p$. (c) $R = 480p - 80p^2$. (d) \$3.00. (e) $P = 492.4p - 80p^2 - 131.4$; \$3.08.

11. 15 times per year, 10.67 each time (11 twice and 10 once, repeated 5 times).

15. (b) -0.009278.

16. –0.625.

18. $R = pq$ is maximum $\Rightarrow \frac{dR}{dp} = q + p\frac{dq}{dp} = 0 \Rightarrow 1 + \frac{p}{q}\frac{dq}{dp} = 0 \Rightarrow E = -1$.

Section 4.7

2. Decreasing at 6π square centimeters per second.

3. Decreasing at $10(3^{1/4})$ square inches per second.

6. $-\frac{1}{4}$ foot per second.

8. 3.75 m/sec.

11. -81.135 ft/sec; 0.045 radians/sec.

12. 8.499 knots

16. Increasing at \$200 per month.

18. constant growth: 2.04 cm in diameter; exponential growth: 3.56 cm in diameter.

Section 4.8

2. Both functions have zeros at 0 and a and critical point at $\frac{a}{2}$; the first function has a minimum of $-\frac{a^2}{4}$ there and the second has a maximum of $\frac{a^2}{4}$ there.

4. The critical points are $\dfrac{-b \pm \sqrt{b^2 - 3ac}}{3a}$, and the average of the critical points is $-\frac{b}{3a}$, which is the inflection point. Thus the inflection point is midway between the critical points.

6. Yes, it will.

8. $y = \frac{1}{16}x^3 - \frac{15}{16}x^2 + \frac{63}{16}x - \frac{81}{16}$.

9. $f(x) = x^3 - ax$ has zeros at 0 and $\pm\sqrt{a}$, extrema at $(-\sqrt{\frac{a}{3}}, \frac{2a}{3}\sqrt{\frac{a}{3}})$ and $(\sqrt{\frac{a}{3}}, -\frac{2a}{3}\sqrt{\frac{a}{3}})$, and inflection point at the origin. The other function has the extrema reversed.

12. $y = \frac{45e}{32}xe^{-\frac{15}{16}x}$.

15. $y = 4(1 - e^{-x\ln 2})$.

16. about 51.6 days.

19. $(\frac{\ln b}{k}, \frac{a}{2})$.

22. about 1557.

Section 4.9

1. (b) $-\frac{1}{9}$. (d) –1. (f) 0. (h) 0. (j) $\frac{1}{3}$. (l) $-\infty$.

2. (b) 0. (d) 1. (f) e. (h) 2. (j) 0. (l) 0.

3. (b) $\frac{5}{4}$. (d) –4.

4. (b) 2. (d) 0.

6. In the first limit, two applications of L'Hôpital's rule yields the original limit. In the second, the limit of the ratio of the derivatives does not exist, but is neither ∞ nor $-\infty$.

Section 4.10

2. (b) $x_2 = 1$, a critical point of f. (d) converges to –2.103803 after 12 steps.

3. (b) x_3 is imaginary.

4. (b) –0.858094. (d) $\pm 4.621134, \pm 1.591144$. (f) 0.352288. (h) 0.876726.

5. (b) –3.586375,0.112681, 2.474194. (d) $\pm 3.605551, \pm 2.449490$, 0.363636. (f) 0.659046. (h) 0, ± 1.895494. (j) –0.451835, –2.932641, –6.357498, –9.394925. (l) 0.680598, 3.141541.