Solutions to Some Problems, *Calculus* (Third Ed.)

Chapter 6

Section 6.1

1. (b) $L_f(4) = \frac{11}{4}; R_f(4) = \frac{13}{4}$. (d) $L_f(n) = [2(1) + 2(1 + \frac{1}{n}) + 2(1 + \frac{2}{n}) + \cdots + 2(1 + \frac{n-1}{n})]\frac{1}{n}$; $R_f(n) = [2(1 + \frac{1}{n}) + 2(1 + \frac{2}{n}) + \cdots + 2(1 + \frac{n}{n})]\frac{1}{n}$. (f) 3.

2. (b) $LHS(n)$ is an overestimate; $RHS(n)$ is an underestimate. (d) 3.395; 2.729.

3. (b) 0.715249; 1.238848. (d) Yes; $L_f(n)$ is an underestimate, so should increase as n increases. $R_f(n)$ should decrease as n increases, because it is an overestimate.

4. (b) $\frac{32}{3}$. (d) $\frac{1}{4}$.

6. 2; 6.

8. (b) $1^2 + 2^2 + \cdots + n^2 + (n+1)^2 = \frac{n(n+1)(2n+1)}{6} + (n+1)^2 = (n+1)[\frac{n(2n+1)}{6} + n + 1] = \frac{(n+1)(n+2)(2n+3)}{6}$. (d) $1^4 + 2^4 + \cdots + n^4 + (n+1)^4 = \frac{n(n+1)(2n+1)(3n^2+3n+1)}{30} + (n+1)^4 = \cdots = \frac{(n+1)(n+2)(2n+3)[3(n+1)^2+3(n+1)+1]}{30}$.

Section 6.2

2. The car travels between 80.6 and 146.6 feet, so probably hits the skunk.

3. Between 0.8630 and 0.9947.

5. (b) between early February and late June. (d) lower.

8. (b) 200; 575. (d) 50. (f) Yes, just past 4:30.

9. (b) Yes. By 10 am, 1400 more have arrived and 2000 have been served, so the line disappears. It forms again at 10:30. (c) at 2 pm; 875; 52.5 min. (d) 3:45 pm. (e) 7850.

10. (b) $\frac{2}{2} + \frac{3}{2} + \frac{4}{2} + \frac{5}{2} + \frac{6}{2} + \frac{7}{2} + \frac{8}{2}$. (d) $e^3 + e^6 + e^9 + \cdots + e^{3n}$.

11. (b) $\sum_{k=1}^{n} \frac{1}{2^k}$. (d) $\sum_{k=1}^{30} 3k$.

12. (d) $\sum_{k=1}^{n}(3k+5) = 3\sum_{k=1}^{n} k+\sum_{k=1}^{n} 5 = 3\frac{n(n+1)}{2}+5n = \frac{1}{2}[3n(n+1)+10n] = \frac{1}{2}n[3(n+1)+10] = \frac{1}{2}n(3n+13)$.

13. The sum is the total change.

14. $\Delta d = s(b) - s(a) = \frac{1}{2}gb^2 - \frac{1}{2}ga^2 = \frac{1}{2}g(b+a)(b-a) = \frac{ga+gb}{2}(b-a) = \frac{v_i+v_f}{2}\Delta t$.

Section 6.3

1. (b) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(-1+2\frac{k}{n}\right)^2\frac{2}{n} = \frac{2}{3}$. (d) $\lim_{n\to\infty}\sum_{k=1}^{n}\left[\left(1+3\frac{k}{n}\right)-\left(1+3\frac{k}{n}\right)^2\right]\frac{3}{n} = -\frac{27}{2}$.

2. (b) 10. (d) 2π. (f) $6+\frac{\pi}{2}$.

3. (b) $\int_0^1 x^3 dx$. (d) $\int_0^\pi \sin x\, dx$.

4. The area of $[n, n+1]\times[0, n]$ is n.

6. The area is $1\cdot(b-a)$.

9. (c) $\delta \le \frac{1}{4}\epsilon$. (d) $\delta \le \frac{1}{6}\epsilon$.

Section 6.4

1. (b) 1.896119; 1.896119; 1.896119; 2.052344; 2.000269. (d) 0.809981; 0.759981; 0.784981; 0.785606; 0.785398. (f) 0.05579; 0.10207; 0.07893; 0.07665; 0.07741.

2. 0.258514, 0.447933; 0.327067, 0.366377; 0.342652, 0.350506. Accuracy is only one or two decimal places.

3. 0.349758, 0.344992; 0.346702, 0.346508; 0.346579, 0.346571; about 5 decimal places.

4. 0.346581, 0.346574, 0.346574; six decimal places.

5. (b) 0.785398 [$SR_f(50)$ and $SR_f(100)$]. (d) 0.535154 [$SR_f(40)$ and $SR_f(80)$]. (f) 0.659330 [$SR_f(30)$ and $SR_f(60)$]. (h) 0.908200178 [$SR_f(40)$ and $SR_f(80)$].

6. $L < T < I < M < R$.

8 (b) Decreasing and concave upward.

14. (b) constant. (d) linear.

15. $\epsilon_i = \frac{1}{2}(Bx_i - Bx_{i-1})(x_i - x_{i-1}) = \frac{1}{2}B(\Delta x)^2; \epsilon = n\epsilon_i = \frac{1}{2}Bn(\frac{b-a}{n})^2 = \frac{B(b-a)^2}{2n}$.

18. (c) $M_f(n) = I - \epsilon$; $M_f(2n) \approx I - \frac{1}{4}\epsilon$; "solve" for I.

Section 6.5

1. (b) $K - J$. (d) $3(K - J)$. (f) $-J$. (h) $8(J - K)$. (j) $7I$. (l) $-5(I + K - J)$.

2. (b) $6Q$. (d) $Q - R$. (f) $R - 4P - 4Q$. (h) $5R + 2Q$. (j) $-4Q + R$. (l) 0.

3. (b) $\bar{f} = 4; z = 3$. (d) $\bar{f} = \frac{\pi}{4}; z = \pm\sqrt{1 - (\frac{\pi}{4})^2}$. (f) $\bar{f} = \frac{4}{5}$; no z because f is not continuous. (h) $\bar{f} = \frac{3}{2}; z = \frac{3}{2}$.

4. (b) $\int_{-1}^{2} x^3 dx = \int_{1}^{2} x^3 dx$. Using, for example, $LHS(2)$ and $RHS(2)$, we get $\frac{275}{64} < \int_{-1}^{2} x^3 dx < \frac{1267}{64}$.

5. (b) $2\int_0^2 (7x^2 + 1)dx$. (d) $2\int_0^8 x^2 dx$. (f) $2\int_0^2 x^2 dx$.

6. (a) $\int_0^n \lfloor x \rfloor \, dx = \int_0^1 \lfloor x \rfloor \, dx + \int_1^2 \lfloor x \rfloor \, dx + \int_2^3 \lfloor x \rfloor \, dx + \cdots + \int_{n-1}^n \lfloor x \rfloor \, dx = 0 + 1 + 2 + \cdots + (n-1) = \frac{(n-1)n}{2}$.

8. $\frac{1}{5}\int_0^5 100e^{-x} dx$.

Section 6.6

1. (b) $\frac{2}{11}$. (d) $\frac{3}{7}(2^7 - 1)$. (f) $\frac{1}{2}(1 - 2^{-4})$. (h) $3(2^{1/3} - 1)$. (j) $\frac{2001}{32}$. (l) 6.

2. (b) $\frac{1}{2}$. (d) $\sqrt{2} - 1$. (f) $\sqrt{2} - \frac{2}{\sqrt{3}}$. (h) $\sqrt{3} - 1$. (j) $\frac{5}{\sqrt{2}} - 1$. (l) $-\frac{1}{3}$.

3. (b) $\frac{2}{\ln 3}$. (d) $\frac{\sqrt{2}-1}{\sqrt{2}\ln 2}$. (f) $\frac{\pi}{4}$. (h) $4 + \ln 3$. (j) $\frac{\pi}{4}$.

4. (b) $x^3 + x^{-2}$. (d) $\frac{1}{x^2+1}$.

6. (b) $\frac{1}{(\sqrt{x}+2)2\sqrt{x}}$. (d) $\frac{1}{2\sqrt{x}} \sin x - 2x \sin x^4$.

9. By the Fundamental Theorem, $\int_{t_1}^{T} v(t)dt = s(T) - s(t_1) = s(T) - s_1$; solve for $s(T)$.

10. (b) $s(10) = -\frac{416}{3}$.

Section 6.7

1. (b) $\frac{3}{7}(1 + 2^{7/3})$. (d) $\frac{3}{4}(6^{4/3} - 2^{4/3})$. (f) $(x^2 + 9)^{3/2} + C$. (h) $\frac{1}{2}\ln(x^2 + 1) + C$. (j) $\frac{10}{9}(1 + x^{3/5})^{3/2} + C$. (l) 7.

2. (b) $-\frac{1}{3}\cos^3 x+C$. (d) $\frac{1}{2}$. (f) $2\sin\sqrt{x}+C$. (h) $\ln|\sec x|+C$. (j) $-\frac{1}{a}\cos ax+C$. (l) $\frac{1}{a}\ln|\sec ax|+C$.

3. (b) $\frac{1}{2}(e^5-e^{-1})$. (d) $-\frac{3}{2}e^{-x^2/3}+C$. (f) $-\frac{1}{3\ln 10}10^{-3x}+C$. (h) $\ln(1+e^x)+C$. (j) $\ln|\ln x|+C$. (l) $\tan^{-1}(e^x)+C$. (n) $\frac{1}{a\ln b}b^{ax}+C$.

4. (b) $\frac{6}{5}(x+5)^{5/2}-10(x+5)^{3/2}+C$. (d) $x-5\ln|x+5|+C$. (f) $\frac{1}{5}(9-x^2)^{5/2}-3(9-x^2)^{3/2}+C$.

5. (b) $y^3=\frac{3}{2}\sin(x^2+1)+C$.

6. (b) $y^3=6(\tan^{-1}x)^2$.

7. (b) $D_x=\frac{1}{a}\cdot\frac{a}{ax+b}=\frac{1}{ax+b}$. (d) $D_x=\dfrac{1}{\sqrt{1-\frac{x^2}{a^2}}}\cdot\dfrac{1}{a}=\dfrac{1}{\sqrt{a^2-x^2}}$. (f) $D_x=\frac{1}{a}\cdot\frac{a\cos ax}{\sin ax}=\cot ax$.

13. Use L'Hôpital.