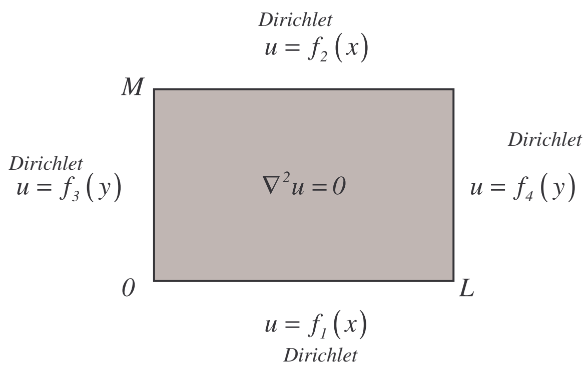
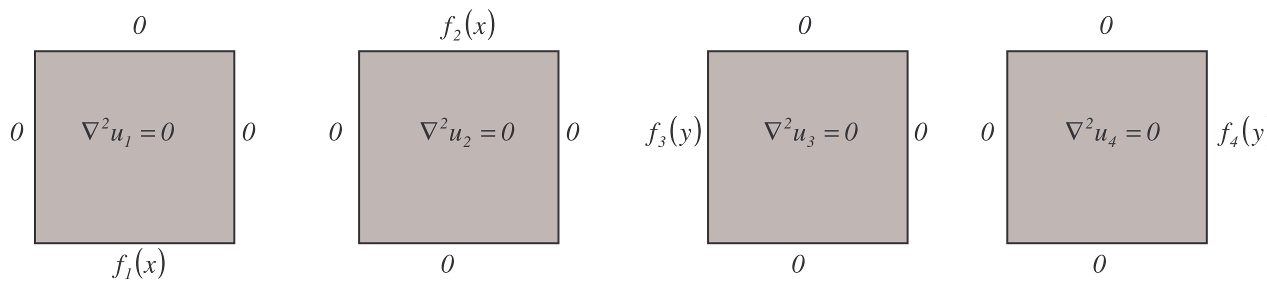
The Laplace Equation: 01 – DDDD (Dirichlet- Dirichlet-Dirichlet-Dirichlet)



Supplemental problems:



Solution of supplemental problems:

$$u_1(x,y) = \sum_{n=1}^{\infty} a_n \sin\frac{n\pi}{L}x \sinh\frac{n\pi}{L}(y-M) \qquad a_n = \frac{-\frac{2}{L}\int_0^L f_1(x)\sin\frac{n\pi}{L}x\,dx}{\sinh\frac{n\pi}{L}M}$$

$$u_2(x,y) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi}{L}x \sinh\frac{n\pi}{L}y \qquad b_n = \frac{\frac{2}{L}\int_0^L f_2(x)\sin\frac{n\pi}{L}x\,dx}{\sinh\frac{n\pi}{L}M}$$

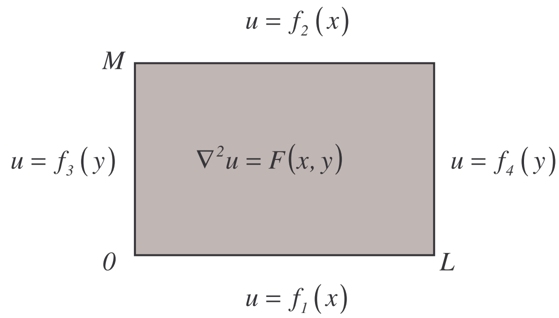
$$u_3(x,y) = \sum_{n=1}^{\infty} c_n \sinh\frac{n\pi}{M}(x-L) \sin\frac{n\pi}{M}y \qquad c_n = \frac{-\frac{2}{M}\int_0^M f_3(y)\sin\frac{n\pi}{M}y\,dy}{\sinh\frac{n\pi}{M}L}$$

$$u_4(x,y) = \sum_{n=1}^{\infty} d_n \sinh\frac{n\pi}{M}x \sin\frac{n\pi}{M}y \qquad d_n = \frac{\frac{2}{M}\int_0^M f_4(y)\sin\frac{n\pi}{M}y\,dy}{\sinh\frac{n\pi}{M}L}$$

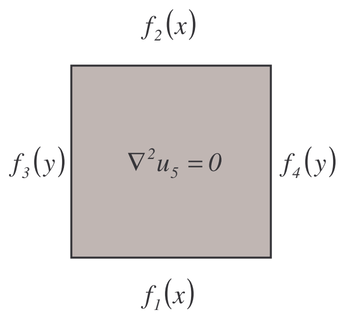
Solution of BVP problem:

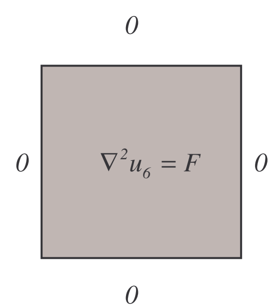
$$u(x,y) = u_1(x,y) + u_2(x,y) + u_3(x,y) + u_4(x,y)$$

POISSON'S EQUATION: 01 – DDDD



Supplemental problems: $\downarrow$





Solution of supplemental problems:

Solution of Laplace's homogeneous equation with non-homogeneous b.c.'s:

$$u_5(x, y) = u_1(x, y) + u_2(x, y) + u_3(x, y) + u_4(x, y)$$

Solution of Poisson's equation with homogeneous boundary conditions

$$u_6(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_{nm} \sin\left(\frac{n\pi}{L} x\right) \sin\left(\frac{m\pi}{M} y\right)$$

$$A_{nm} = \frac{-4}{\pi^2 LM \left(\frac{n^2}{L^2} + \frac{m^2}{M^2}\right)} \int_0^M \int_0^L F(x, y) \sin\left(\frac{n\pi}{L} x\right) \sin\left(\frac{m\pi}{M} y\right) dxdy$$

Solution of BVP for Poisson's Equation (superposition principle):

$$u(x, y) = u_5(x, y) + u_6(x, y)$$