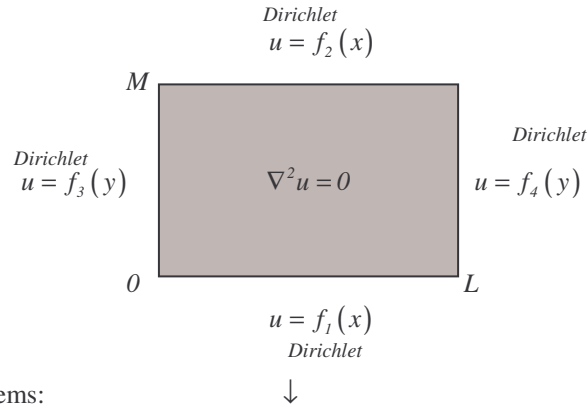
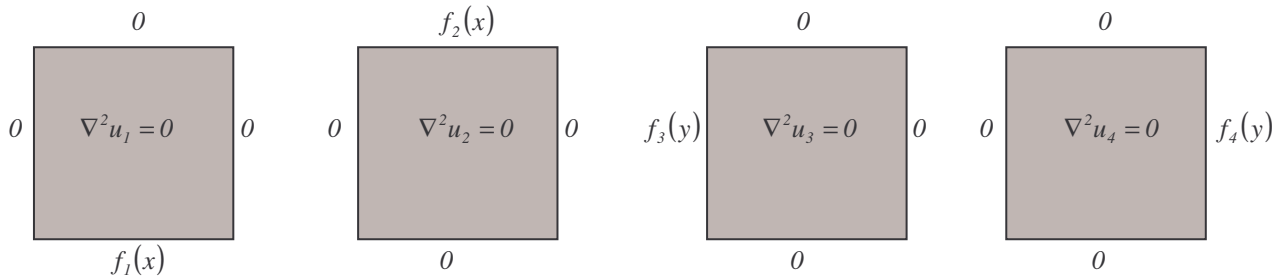


The Laplace Equation: 01 – DDDD (Dirichlet- Dirichlet-Dirichlet-Dirichlet)

Supplemental problems:



Solution of supplemental problems:

$$u_1(x, y) = \sum_{n=1}^{\infty} a_n \sin \frac{n\pi}{L} x \sinh \frac{n\pi}{L} (y - M)$$

$$a_n = \frac{-\frac{2}{L} \int_0^L f_1(x) \sin \frac{n\pi}{L} x dx}{\sinh \frac{n\pi}{L} M}$$

$$u_2(x, y) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{L} x \sinh \frac{n\pi}{L} y$$

$$b_n = \frac{\frac{2}{L} \int_0^L f_2(x) \sin \frac{n\pi}{L} x dx}{\sinh \frac{n\pi}{L} M}$$

$$u_3(x, y) = \sum_{n=1}^{\infty} c_n \sinh \frac{n\pi}{M} (x - L) \sin \frac{n\pi}{M} y$$

$$c_n = \frac{-\frac{2}{M} \int_0^M f_3(y) \sin \frac{n\pi}{M} y dy}{\sinh \frac{n\pi}{M} L}$$

$$u_4(x, y) = \sum_{n=1}^{\infty} d_n \sinh \frac{n\pi}{M} x \sin \frac{n\pi}{M} y$$

$$d_n = \frac{\frac{2}{M} \int_0^M f_4(y) \sin \frac{n\pi}{M} y dy}{\sinh \frac{n\pi}{M} L}$$

Solution of BVP problem:

$$u(x, y) = u_1(x, y) + u_2(x, y) + u_3(x, y) + u_4(x, y)$$

POISSON'S EQUATION: 01 – DDDD

A rectangular domain with vertices labeled 0 (bottom-left), L (bottom-right), M (top-left), and 0 (top-right). The boundary conditions are: $u = f_2(x)$ on the top edge, $u = f_4(y)$ on the right edge, $u = f_1(x)$ on the bottom edge, and $u = f_3(y)$ on the left edge. Inside the domain, the equation is $\nabla^2 u = F(x, y)$.

Supplemental problems:



Two rectangular domains are shown side-by-side. The left domain has boundary conditions $f_2(x)$ (top), $f_4(y)$ (right), $f_1(x)$ (bottom), and $f_3(y)$ (left), with the equation $\nabla^2 u_5 = 0$ inside. The right domain has boundary conditions 0 on all four edges and the equation $\nabla^2 u_6 = F$ inside.

Solution of supplemental problems:

Solution of Laplace's homogeneous equation with non-homogeneous b.c.'s:

$$u_5(x, y) = u_1(x, y) + u_2(x, y) + u_3(x, y) + u_4(x, y)$$

Solution of Poisson's equation with homogeneous boundary conditions

$$u_6(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_{nm} \sin\left(\frac{n\pi}{L}x\right) \sin\left(\frac{m\pi}{M}y\right)$$

$$A_{nm} = \frac{-4}{\pi^2 LM \left(\frac{n^2}{L^2} + \frac{m^2}{M^2}\right)} \int_0^L \int_0^M F(x, y) \sin\left(\frac{n\pi}{L}x\right) \sin\left(\frac{m\pi}{M}y\right) dx dy$$

Solution of BVP for Poisson's Equation (superposition principle):

$$u(x, y) = u_5(x, y) + u_6(x, y)$$