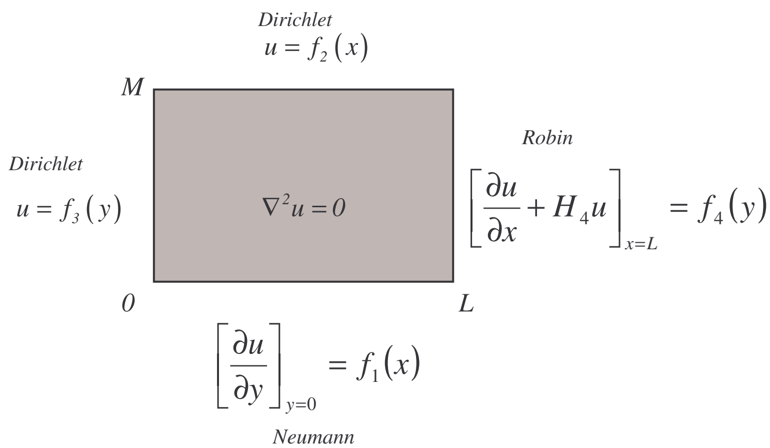
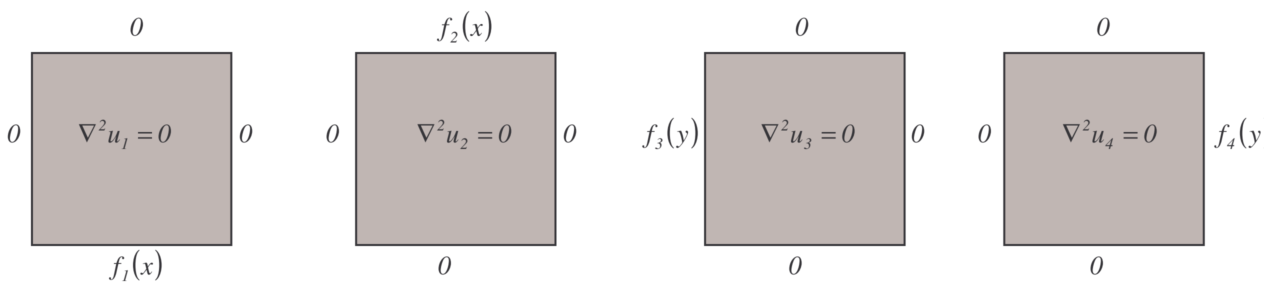
The Laplace Equation: 02 – NDDR (Neumann-Dirichlet-Dirichlet-Robin)



Supplemental problems: ↓



Solution of supplemental problems:

$$u_1(x,y) = \sum_{n=1}^{\infty} a_n \sin\lambda_n x \sinh[\lambda_n(y-M)] \qquad a_n = \frac{\int_0^L f_1(x)\sin\lambda_n x\,dx}{\left[\frac{L}{2} - \frac{\sin 2\lambda_n L}{4\lambda_n}\right]\lambda_n \cosh\lambda_n M}$$

$$\lambda_n \to positive\_roots\_of : \lambda\cos\lambda L + H_4 \sin\lambda L = 0$$

$$u_2(x,y) = \sum_{n=1}^{\infty} b_n \sin\lambda_n x \cosh\lambda_n y \qquad b_n = \frac{\int_0^L f_2(x)\sin\lambda_n x\,dx}{\left[\frac{L}{2} - \frac{\sin 2\lambda_n L}{4\lambda_n}\right]\cosh\lambda_n M}$$

$$\lambda_n \to positive\_roots\_of : \lambda\cos\lambda L + H_4 \sin\lambda L = 0$$

$$u_3(x,y) = \sum_{n=1}^{\infty} c_n \left\{\cosh[\lambda_n(x-L)] - \frac{H_4}{\lambda_n}\sinh[\lambda_n(x-L)]\right\}\cos\lambda_n y$$

$$\lambda_n = \left(n + \frac{1}{2}\right)\frac{\pi}{M} \qquad c_n = \frac{\frac{2}{M}\int_0^M f_3(y)\cos\lambda_n y\,dy}{\cosh\lambda_n L + \frac{H_4}{\lambda_n}\sinh\lambda_n L}$$

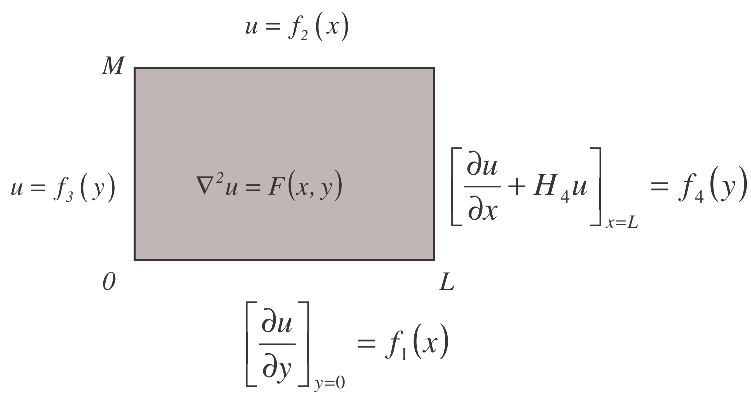
$$u_4(x,y)=\sum_{n=1}^{\infty} d_n \sinh \lambda_n x \cos \lambda_n y \qquad d_n = \frac{\frac{2}{M}\int_0^M f_4(y)\cos \lambda_n y dy}{\lambda_n \cosh \lambda_n L + H_4 \sinh \lambda_n L}$$

$$\lambda_n = \left(n+\frac{1}{2}\right)\frac{\pi}{M}$$

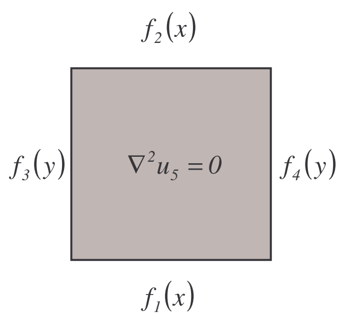
Solution of BVP problem:

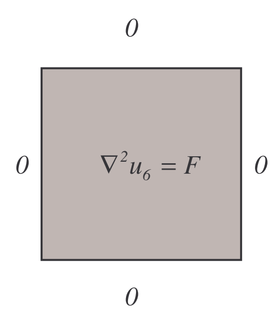
$$u(x,y)=u_1(x,y)+u_2(x,y)+u_3(x,y)+u_4(x,y)$$

POISSON'S EQUATION: 02 – NDDR



Supplemental problems: ↓





Solution of supplemental problems:

Solution of Laplace's homogeneous equation with non-homogeneous b.c.'s:

$$u_5(x,y)=u_1(x,y)+u_2(x,y)+u_3(x,y)+u_4(x,y)$$

Solution of Poisson's equation with homogeneous boundary conditions

$$u_6(x,y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{nm} \sin\lambda_n x \cos\mu_m y$$

$$A_{nm} = \frac{-1}{\frac{M}{2}\left[\frac{L}{2} - \frac{\sin(2\lambda_n L)}{4\lambda_n}\right](\lambda_n^2 + \mu_m^2)} \int_0^M\int_0^L F(x,y)\sin(\lambda_n x)\cos(\mu_m y)dxdy$$

$$\lambda_n \to positive\_roots\_of : \lambda\cos\lambda L + H_4 \sin\lambda L = 0 \qquad \mu_m = \left(m + \frac{1}{2}\right)\frac{\pi}{M}$$

Solution of BVP for Poisson's Equation (superposition principle):

$$u(x,y) = u_5(x,y) + u_6(x,y)$$