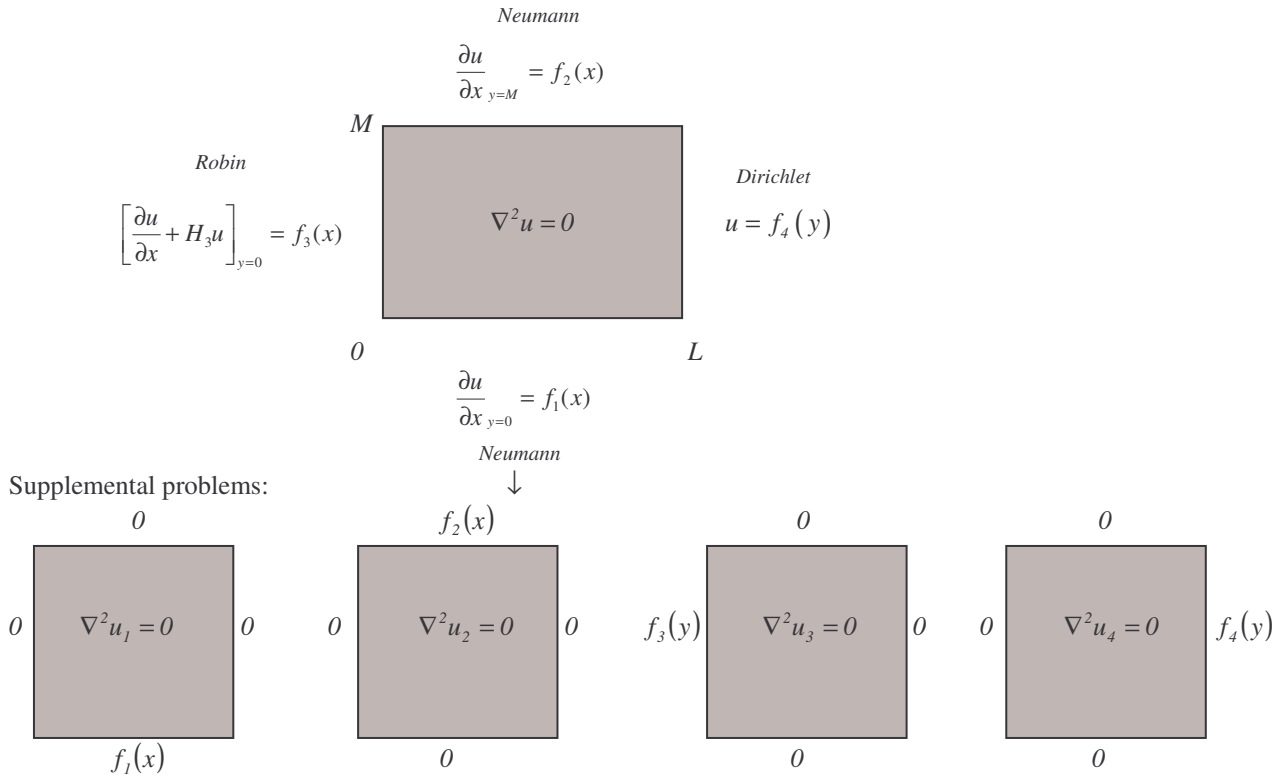


The Laplace Equation: 03 – NNRD (Neumann-Neumann-Robin-Dirichlet)Solution of supplemental problems:

$u_1(x, y) = \sum_{n=1}^{\infty} [a_n \cdot \sin[\lambda_n \cdot (x - L)] \cdot \cosh[(y - M) \cdot \lambda_n]]$	$-\int_0^L f_1 \cdot \sin[\lambda_n \cdot (x - L)] dx$ $a_n = \frac{\left(\frac{L}{2} - \frac{\sin(2\lambda_n \cdot L)}{4\lambda_n} \right) \cdot \lambda_n \cdot \sinh(\lambda_n \cdot M)}{\lambda_n \cdot \cosh(\lambda_n \cdot L) + H_3 \cdot \sin(\lambda_n \cdot L)} = 0$	$\lambda_n \cdot \cosh(\lambda_n \cdot L) + H_3 \cdot \sin(\lambda_n \cdot L) = 0$
$u_2(x, y) = \sum_{n=1}^{\infty} [b_n \cdot \sin[\lambda_n \cdot (x - L)] \cdot \cosh(\lambda_n \cdot y)]$	$\int_0^L f_2 \cdot \sin[\lambda_n \cdot (x - L)] dx$ $b_n = \frac{\left(\frac{L}{2} - \frac{\sin(2\lambda_n \cdot L)}{4\lambda_n} \right) \cdot \lambda_n \cdot \sinh(\lambda_n \cdot M)}{\lambda_n \cdot \cosh(\lambda_n \cdot L) + H_3 \cdot \sin(\lambda_n \cdot L)} = 0$	$\lambda_n \cdot \cosh(\lambda_n \cdot L) + H_3 \cdot \sin(\lambda_n \cdot L) = 0$
$u_3(x, y) = c_0 \cdot (x - L) + \sum_{n=1}^{\infty} [c_n \cdot \sinh[\lambda_n \cdot (x - L)] \cdot \cos(\lambda_n \cdot y)]$	$\frac{2}{M} \int_0^M f_3 \cdot \cos(\lambda_n \cdot y) dy$ $c_n = \frac{\int_0^M f_3 dy}{\lambda_n \cdot \cosh(\lambda_n \cdot L) - H_3 \cdot \sinh(\lambda_n \cdot L)}$ $c_0 = \frac{1}{M \cdot (1 - H_3 \cdot L)}$	$\lambda_n = \frac{n \cdot \pi}{M}$
$u(x, y) = d_0 \cdot \left(x + \frac{1}{H_3} \right) + \sum_{n=1}^{\infty} \left[d_n \cdot \left(\cosh(\lambda_n \cdot x) + \frac{H_3}{\lambda_n} \cdot \sinh(\lambda_n \cdot x) \right) \cdot \cos(\lambda_n \cdot y) \right]$	$\frac{2}{M} \int_0^M f_4 \cdot \cos(\lambda_n \cdot y) dy$ $d_n = \frac{\int_0^M f_4 dy}{\cosh(\lambda_n \cdot L) + \frac{H_3}{\lambda_n} \cdot \sinh(\lambda_n \cdot L)}$ $d_0 = \frac{1}{\left(L + \frac{1}{H_3} \right) \cdot M}$	$\lambda_n = \frac{n \cdot \pi}{M}$

Solution of BVP problem:

$$u(x, y) = u_1(x, y) + u_2(x, y) + u_3(x, y) + u_4(x, y)$$

POISSON'S EQUATION: 03 – NNRD

$$\begin{array}{c}
 u = f_2(x) \\
 \begin{array}{ccc}
 M & & \\
 u = f_3(y) & \nabla^2 u = F(x, y) & u = f_4(y) \\
 0 & & L \\
 & u = f_1(x) &
 \end{array}
 \end{array}$$

Supplemental problems:

↓

$$\begin{array}{cc}
 \begin{array}{c} f_2(x) \\ \begin{array}{ccc} & & \\ f_3(y) & \nabla^2 u_5 = 0 & f_4(y) \\ & & \\ f_1(x) & & \end{array} \end{array} & \begin{array}{c} 0 \\ \begin{array}{ccc} & & \\ 0 & \nabla^2 u_6 = F & 0 \\ & & \\ 0 & & \end{array} \end{array}
 \end{array}$$

Solution of supplemental problems:

Solution of Laplace's homogeneous equation with non-homogeneous b.c.'s:

$$u_5(x, y) = u_1(x, y) + u_2(x, y) + u_3(x, y) + u_4(x, y)$$

Solution of Poisson's equation with homogeneous boundary conditions

$$u_6(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} [A_{nm} \cdot \cos(\lambda_n \cdot y) \cdot \sin[\lambda_m \cdot (x - L)]]$$

$$A_{nm} = \frac{\frac{-2}{M} \cdot \int_0^L \int_0^M F(x, y) \cdot \cos(\lambda_m \cdot y) \cdot \sin[\lambda_n \cdot (x - L)] \, dy \, dx}{\left(\frac{L}{2} - \frac{\sin(2\lambda_n \cdot L)}{4\lambda_n} \right) \cdot (\lambda_n^2 + \lambda_m^2)}$$

$$\lambda_m = \frac{m \cdot \pi}{M}$$

$$\lambda_n \cdot \cos(\lambda_n \cdot L) + H_3 \cdot \sin(\lambda_n \cdot L) = 0$$

Solution of BVP for Poisson's Equation (superposition principle):

$$u(x, y) = u_5(x, y) + u_6(x, y)$$