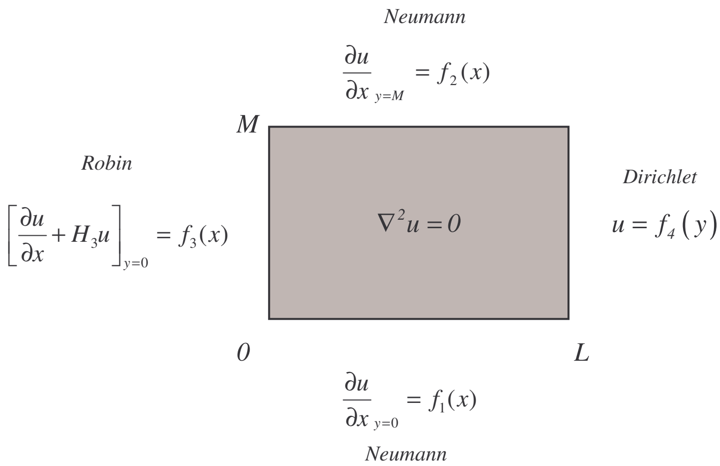
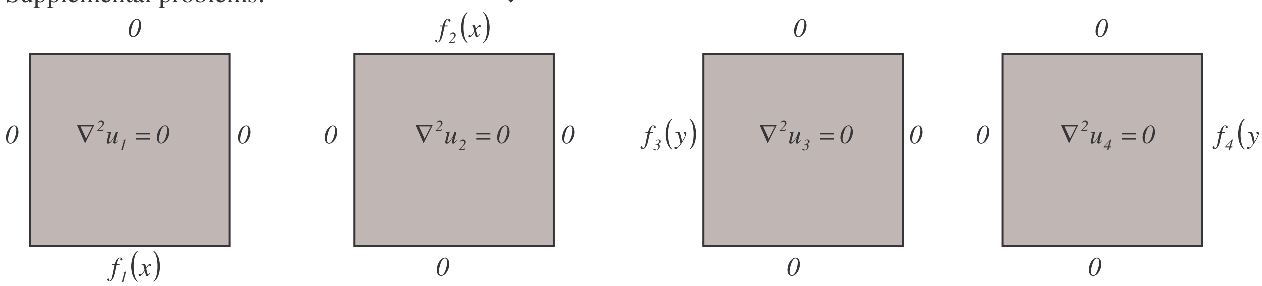
The Laplace Equation: 03 – NNRD (Neumann-Neumann-Robin-Dirichlet)



Supplemental problems:



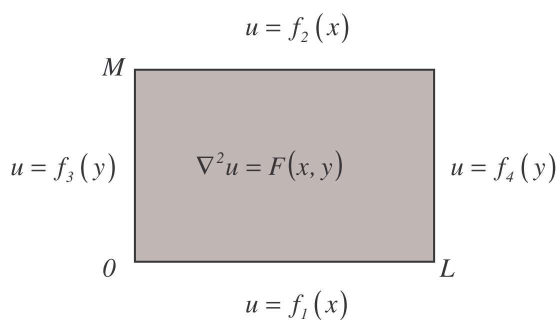
Solution of supplemental problems:

$u_1(x,y) = \sum_{n=1}^{\infty} \left[a_n \cdot \sin[\lambda_n \cdot (x-L)] \cdot \cosh[(y-M) \cdot \lambda_n]\right]$	$a_n = \dfrac{-\int_0^L f_1 \cdot \sin[\lambda_n \cdot (x-L)]\,dx}{\left(\frac{L}{2} - \frac{\sin(2\lambda_n \cdot L)}{4\lambda_n}\right) \cdot \lambda_n \cdot \sinh(\lambda_n \cdot M)}$	$\lambda_n \cdot \cos(\lambda_n \cdot L) + H_3 \cdot \sin(\lambda_n \cdot L) = 0$
$u_2(x,y) = \sum_{n=1}^{\infty} \left[b_n \cdot \sin[\lambda_n \cdot (x-L)] \cdot \cosh(\lambda_n \cdot y)\right]$	$b_n = \dfrac{\int_0^L f_2 \cdot \sin[\lambda_n \cdot (x-L)]\,dx}{\left(\frac{L}{2} - \frac{\sin(2\lambda_n \cdot L)}{4\lambda_n}\right) \cdot \lambda_n \cdot \sinh(\lambda_n \cdot M)}$	$\lambda_n \cdot \cos(\lambda_n \cdot L) + H_3 \cdot \sin(\lambda_n \cdot L) = 0$
$u_3(x,y) = c_0 \cdot (x-L) + \sum_{n=1}^{\infty} \left[c_n \cdot \sinh[\lambda_n \cdot (x-L)] \cdot \cos(\lambda_n \cdot y)\right]$	$c_n = \dfrac{\frac{2}{M}\int_0^M f_3 \cdot \cos(\lambda_n \cdot y)\,dy}{\lambda_n \cdot \cosh(\lambda_n \cdot L) - H_3 \cdot \sinh(\lambda_n \cdot L)}$ $c_0 = \dfrac{\int_0^M f_3\,dy}{M \cdot (1 - H_3 \cdot L)}$	$\lambda_n = \frac{n \cdot \pi}{M}$
$u(x,y) = d_0 \left(x + \frac{1}{H_3}\right) + \sum_{n=1}^{\infty} \left[d_n \cdot \left(\cosh(\lambda_n \cdot x) + \frac{H_3}{\lambda_n} \cdot \sinh(\lambda_n \cdot x)\right) \cdot \cos(\lambda_n \cdot y)\right]$	$d_n = \dfrac{\frac{2}{M}\int_0^M f_4 \cdot \cos(\lambda_n \cdot y)\,dy}{\cosh(\lambda_n \cdot L) + \frac{H_3}{\lambda_n} \cdot \sinh(\lambda_n \cdot L)}$ $d_0 = \dfrac{1}{\left(L + \frac{1}{H_3}\right) \cdot M}\int_0^M f_4\,dy$	$\lambda_n = \frac{n \cdot \pi}{M}$

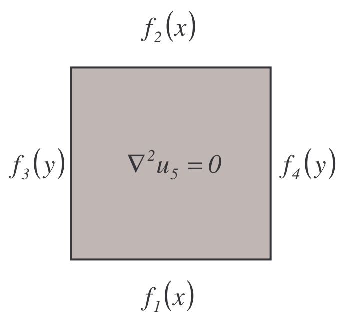
Solution of BVP problem:

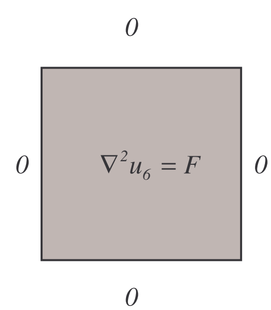
$$u(x,y) = u_1(x,y) + u_2(x,y) + u_3(x,y) + u_4(x,y)$$

POISSON'S EQUATION: 03 – NNRD



Supplemental problems: ↓





Solution of supplemental problems:

Solution of Laplace's homogeneous equation with non-homogeneous b.c.'s:

$$u_5(x,y) = u_1(x,y) + u_2(x,y) + u_3(x,y) + u_4(x,y)$$

Solution of Poisson's equation with homogeneous boundary conditions

$$u_6(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[A_{nm} \cdot \cos(\lambda_n \cdot y) \cdot \sin\left[\lambda_m \cdot (x - L)\right]\right]$$

$$A_{nm} = \frac{\dfrac{-2}{M} \cdot \displaystyle\int_0^L \int_0^M F(x,y) \cdot \cos(\lambda_m \cdot y) \cdot \sin\left[\lambda_n \cdot (x - L)\right] dy\, dx}{\left(\dfrac{L}{2} - \dfrac{\sin(2 \cdot \lambda_n \cdot L)}{4 \cdot \lambda_n}\right) \cdot \left(\lambda_n^{\,2} + \lambda_m^{\,2}\right)}$$

$$\lambda_m = \frac{m \cdot \pi}{M}$$

$$\lambda_n \cdot \cos(\lambda_n \cdot L) + H_3 \cdot \sin(\lambda_n \cdot L) = 0$$

Solution of BVP for Poisson's Equation (superposition principle):

$$u(x,y) = u_5(x,y) + u_6(x,y)$$