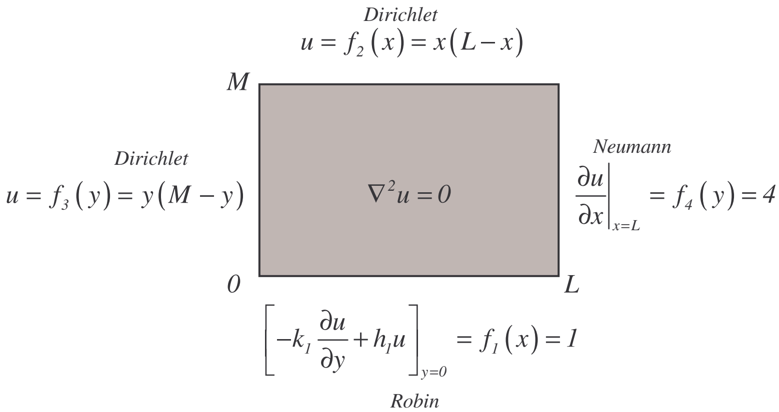
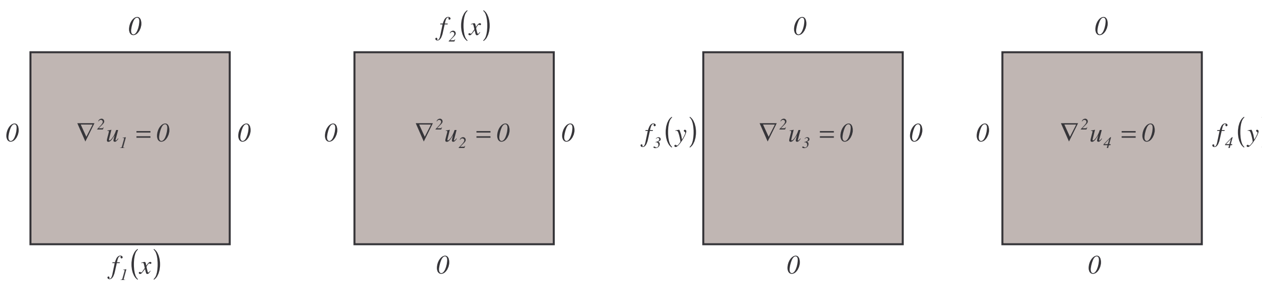
The Laplace Equation: 05 – RDDN (Robin- Dirichlet-Dirichlet-Neumann)



Supplemental problems: ↓



Solution of supplemental problems:

$$u_1(x,y) = \sum_{n=0}^{\infty} a_n \sin(\lambda_n x)\sinh(\lambda_n(y-M)) \qquad a_n = \frac{-\frac{2}{L}\int_0^L f_1(x)\sin(\lambda_n x)\,dx}{k_1\lambda_n \cosh(\lambda_n M) + h_1 \sinh(\lambda_n M)}$$

Where $\lambda_n = \left(n+\frac{1}{2}\right)\frac{\pi}{L}$

$$u_2(x,y) = \sum_{n=0}^{\infty} b_n \sin(\lambda_n x)\left[\cosh(\lambda_n y) + \frac{h_1}{k_1\lambda_n}\sinh(\lambda_n y)\right] \quad b_n = \frac{\frac{2}{L}\int_0^L f_2(x)\sin(\lambda_n x)\,dx}{\cosh(\lambda_n M) + \frac{h_1}{k_1\lambda_n}\sinh(\lambda_n M)}$$

Where $\lambda_n = \left(n+\frac{1}{2}\right)\frac{\pi}{L}$

$$u_3(x,y) = \sum_{n=1}^{\infty} c_n \sin(\lambda_n(y-M))\cosh(\lambda_n(x-L)) \qquad c_n = \frac{\int_0^M f_3(y)\sin(\lambda_n(y-M))\,dy}{\left(\frac{M}{2} - \frac{\sin(2\lambda_n M)}{4\lambda_n}\right)\cosh(\lambda_n L)}$$

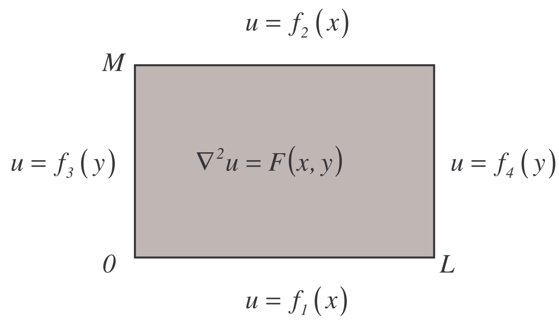
Where λ_n are the positive roots of $\lambda\cos(\lambda M) + H_1\sin(\lambda M) = 0$

$$u_4(x,y) = \sum_{n=1}^{\infty} d_n \sinh\frac{n\pi}{M}x \sin\frac{n\pi}{M}y \qquad d_n = \frac{\frac{2}{M}\int_0^M f_4(y)\sin\frac{n\pi}{M}y\,dy}{\sinh\frac{n\pi}{M}L}$$

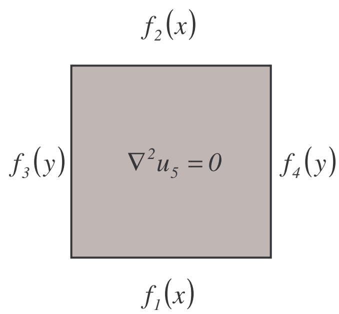
Solution of BVP problem:

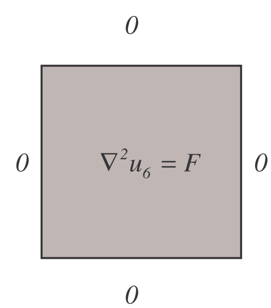
$$u(x,y) = u_1(x,y) + u_2(x,y) + u_3(x,y) + u_4(x,y)$$

POISSON'S EQUATION: 01 – DDDD



Supplemental problems: ↓





Solution of supplemental problems:

Solution of Laplace's homogeneous equation with non-homogeneous b.c.'s:

$$u_5(x,y) = u_1(x,y) + u_2(x,y) + u_3(x,y) + u_4(x,y)$$

Solution of Poisson's equation with homogeneous boundary conditions

$$u_6(x,y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{nm} \sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{M}y\right)$$

$$A_{nm} = \frac{-4}{\pi^2 LM\left(\frac{n^2}{L^2}+\frac{m^2}{M^2}\right)}\int_0^M\int_0^L F(x,y)\sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{M}y\right)dxdy$$

Solution of BVP for Poisson's Equation (superposition principle):

$$u(x,y) = u_5(x,y) + u_6(x,y)$$