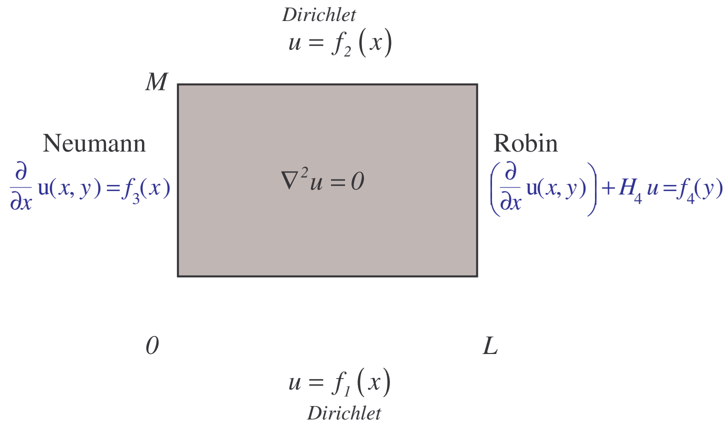
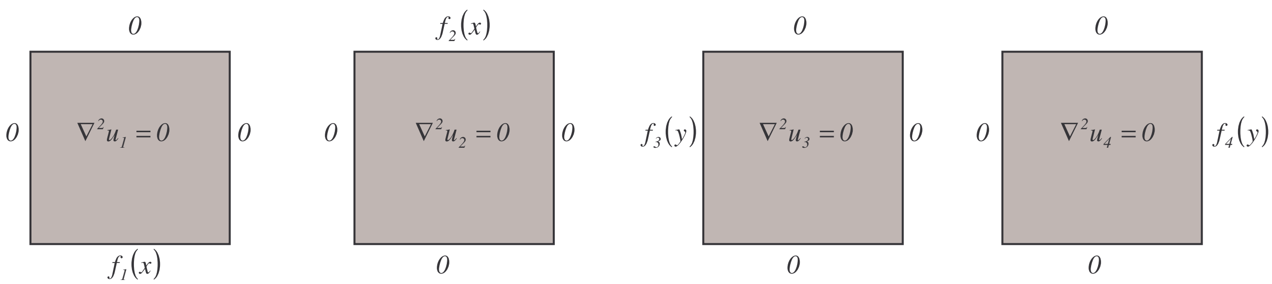
The Laplace Equation: 06 – DDNR (Dirichlet- Dirichlet-Neumann-Robin)



Supplemental problems: ↓



Solution of supplemental problems:

$$u_1(x,y) = \sum_{n=1}^{\infty} a_n \cos(\lambda_n x) \sinh \lambda_n (y - M)$$

$$a_n = \frac{-\int_0^L f_1(x) \cos \lambda_n x dx}{\left(\frac{L}{2} + \frac{\sin 2\lambda_n L}{4\lambda_n}\right) \sinh \lambda_n M}$$

$$\lambda_n \rightarrow positive\_roots\_of : \lambda \sin \lambda L - H_4 \cos \lambda L = 0$$

$$u_2(x,y) = \sum_{n=1}^{\infty} b_n \cos \lambda_n x \sinh \lambda_n y$$

$$b_n = \frac{\int_0^L f_2(x) \cos \lambda_n x dx}{\left(\frac{L}{2} + \frac{\sin 2\lambda_n L}{4\lambda_n}\right) \sinh \lambda_n M}$$

$$\lambda_n \rightarrow positive\_roots\_of : \lambda \sin \lambda L - H_4 \cos \lambda L = 0$$

$$u_3(x,y) = \sum_{n=1}^{\infty} c_n \sin \frac{n\pi}{M} y \left[\cosh \frac{n\pi}{M}(x - L) - \frac{H_4}{\frac{n\pi}{M}} \sinh \frac{n\pi}{M}(x - L)\right]$$

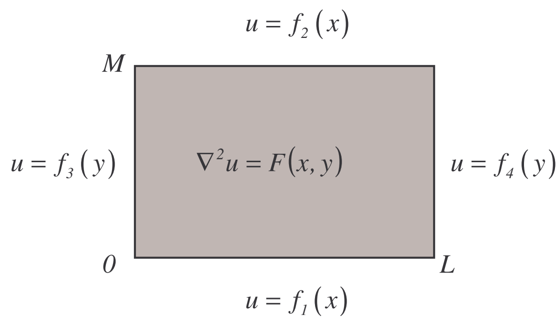
$$c_n = \frac{-\frac{2}{M}\int_0^M f_3(y) \sin \frac{n\pi}{M} y dy}{\frac{n\pi}{M} \sinh \frac{n\pi}{M} L + H_4 \cosh \frac{n\pi}{M} L}$$

$$u_4(x,y)=\sum_{n=1}^{\infty} d_n \sin\frac{n\pi}{M} y \cosh\frac{n\pi}{M} x \qquad d_n=\frac{\frac{2}{M}\int_0^M f_4(y)\sin\frac{n\pi}{M} y dy}{\frac{n\pi}{M}\sinh\frac{n\pi}{M} L + H_4\cosh\frac{n\pi}{M} L}$$

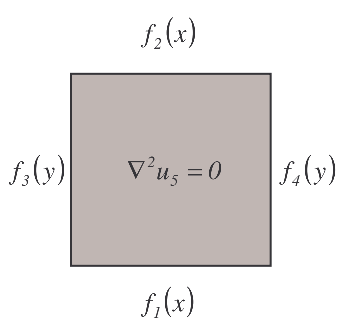
Solution of BVP problem:

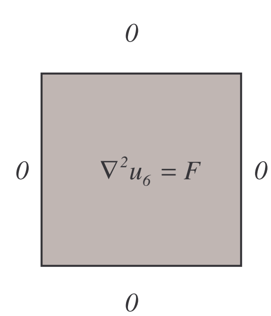
$$u(x,y)=u_1(x,y)+u_2(x,y)+u_3(x,y)+u_4(x,y)$$

POISSON'S EQUATION: 01 – DDDD



Supplemental problems: ↓





Solution of supplemental problems:

Solution of Laplace's homogeneous equation with non-homogeneous b.c.'s:

$$u_5(x,y)=u_1(x,y)+u_2(x,y)+u_3(x,y)+u_4(x,y)$$

Solution of Poisson's equation with homogeneous boundary conditions

$$u(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}\sin\frac{n\pi}{M}y\cos\lambda_m x \qquad \lambda_m \rightarrow positive\_roots\_of: \lambda\sin\lambda L - H_4\cos\lambda L = 0$$

$$A_{mn}=\frac{-\frac{2}{M}}{[\lambda_m{}^2+(\frac{n\pi}{M})^2](\frac{L}{2}+\frac{\sin 2\lambda_n L}{4\lambda_n})}\int_0^M\int_0^L F(x,y)\sin\frac{n\pi}{M}y\cos\lambda_m x dx dy$$

<u>Solution of BVP for Poisson's Equation</u> (superposition principle):

$$u(x,y)=u_5(x,y)+u_6(x,y)$$