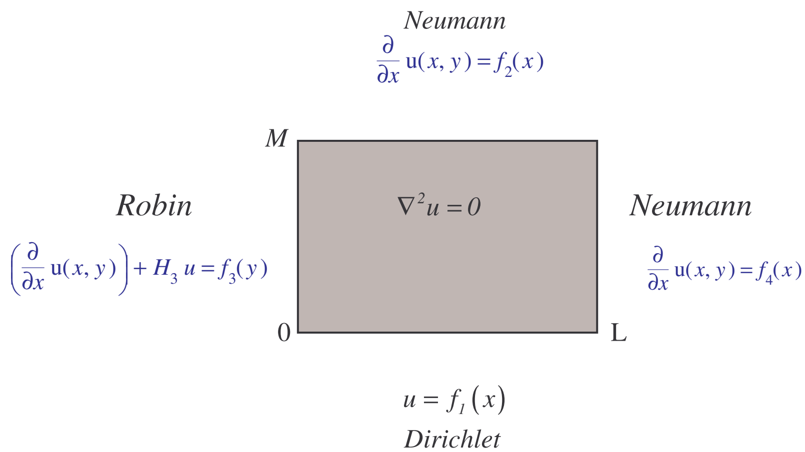
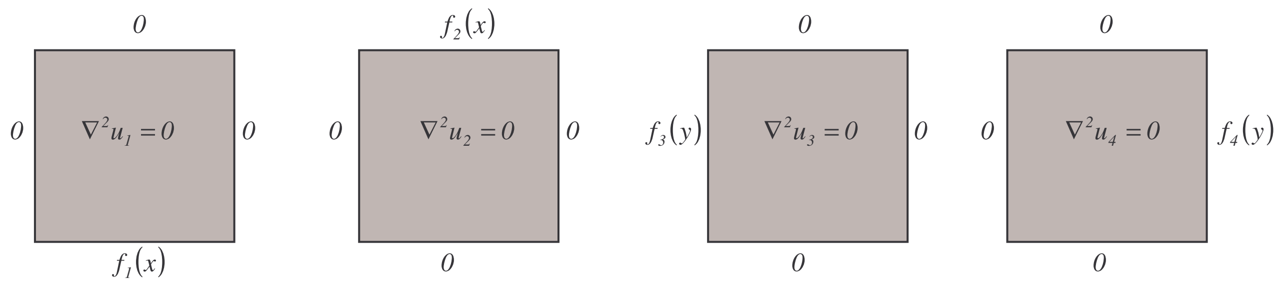
The Laplace Equation: 07 – DNRN (Dirichlet- Neumann-Robin- Neumann)



Supplemental problems:



Solution of supplemental problems:

$$u_1(x,y)=\sum_{n=1}^{\infty} a_n\cos\lambda_n(x-L)\cosh\lambda_n(y-M)$$

$$a_n=\frac{\int_0^L f_1(x)\cos\lambda_n(x-L)dx}{\left(\frac{L}{2}+\frac{\sin 2\lambda_n L}{4\lambda_n}\right)\cosh\lambda_n M}$$

$$\lambda_n \to positive\_roots\_of:\lambda\sin\lambda L-H_3\cos\lambda L=0$$

$$u_2(x,y)=\sum_{n=1}^{\infty} b_n\cos\lambda_n(x-L)\sinh\lambda_n y$$

$$b_n=\frac{\int_0^L f\,2(x)\cos\lambda_n(x-L)dx}{\lambda_n\left(\frac{L}{2}+\frac{\sin 2\lambda_n L}{4\lambda_n}\right)\cosh\lambda_n M}$$

$$\lambda_n \to positive\_roots\_of:\lambda\sin\lambda L-H_3\cos\lambda L=0$$

$$u_3(x,y)=\sum_{n=1}^{\infty} c_n\cosh\lambda_n(x-L)\sin\lambda_n y$$

$$c_n=\frac{-\frac{2}{M}\int_0^M f_3(y)\sin\lambda_n y\,dy}{\lambda_n\sinh\lambda_n L-H_3\cosh\lambda_n L}$$

$$\lambda_n=(n+1/2)\frac{\pi}{M}$$

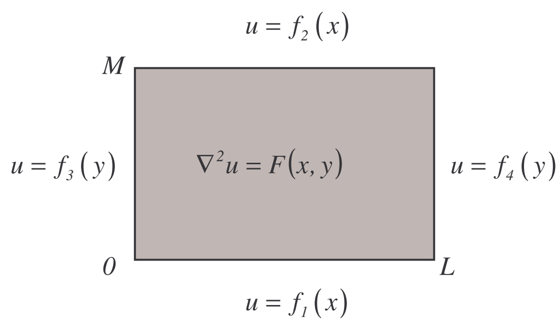
$$u_4(x,y) = \sum_{n=1}^{\infty} d_n \sin\lambda_n y[\cosh\lambda_n x - \frac{H_3}{\lambda_n}\sinh\lambda_n x] \qquad d_n = \frac{\frac{2}{M}\int_0^M f4(y)\sin\lambda_n y dy}{\lambda_n \sinh\lambda_n L - H_3\cosh\lambda_n L}$$

$$\lambda_n = (n+1/2)\frac{\pi}{M}$$

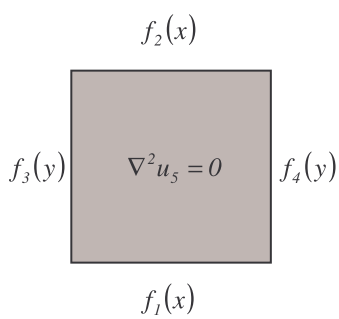
Solution of BVP problem:

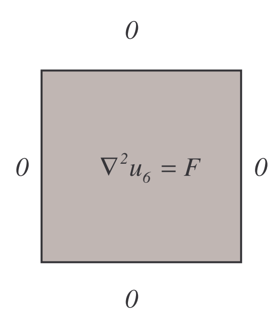
$$u(x,y) = u_1(x,y) + u_2(x,y) + u_3(x,y) + u_4(x,y)$$

POISSON'S EQUATION: 01 – DDDD



Supplemental problems: ↓





Solution of supplemental problems:

Solution of Laplace's homogeneous equation with non-homogeneous b.c.'s:

$$u_5(x,y) = u_1(x,y) + u_2(x,y) + u_3(x,y) + u_4(x,y)$$

Solution of Poisson's equation with homogeneous boundary conditions

$$u(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}\sin\lambda_m y\cos\lambda_n(x-L) \qquad \lambda_m=(n+1/2)\frac{\pi}{M}$$

$$A_{mn}=\frac{-\frac{2}{M}}{[\lambda_m{}^2+(\lambda_n)^2](\frac{L}{2}+\frac{\sin 2\lambda_n L}{4\lambda_n})}\int_0^M\int_0^L F(x,y)\sin\lambda_m y\cos\lambda_n(x-L)dxdy$$

$$\lambda_n \rightarrow positive\_roots\_of:\lambda\sin\lambda L-H_3\cos\lambda L=0$$

<u>Solution of BVP for Poisson's Equation</u> (superposition principle):

$$u(x,y)=u_5(x,y)+u_6(x,y)$$