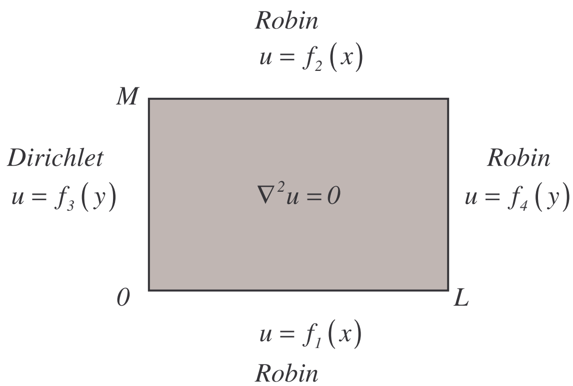
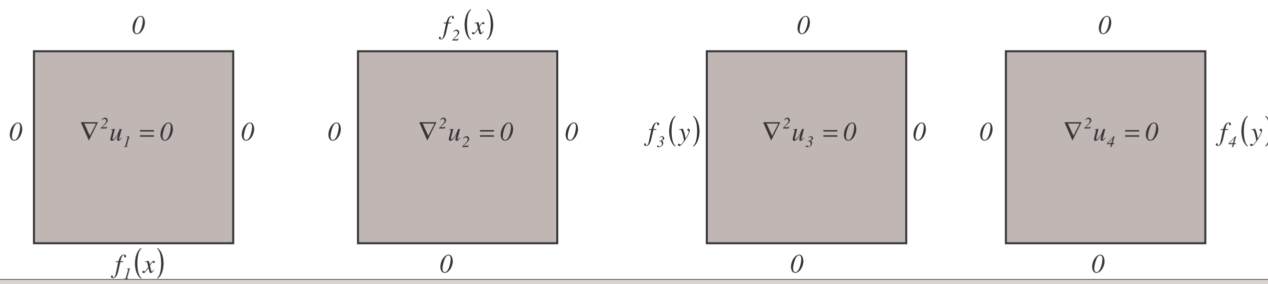
**The Laplace Equation: 11 – RRDR (Robin- Robin-Dirichlet-Robin)**




Supplemental problems: ↓




Solution of supplemental problems:

$$u_1(x,y)=\sum_{n=1}^{\infty} a_n \sin(\lambda_n x)\left[\cosh[\lambda_n(y-M)]-\frac{H_2}{\lambda_n}\sinh[\lambda_n(y-M)]\right]$$

$$a_n=\frac{\int_0^L f_1(x)\sin(\lambda_n x)dx}{\left[\frac{L}{2}-\frac{\sin(2\lambda_n)}{4\lambda_n}\right]\left[\lambda_n \sinh(\lambda_n M)+H_2\cosh(\lambda_n M)+H_1\cosh(\lambda_n M)\right]}$$

$$u_2(x,y)=\sum_{n=1}^{\infty} b_n \sin(\lambda_n x)\left[\cosh(\lambda_n y)-\frac{H_1}{\lambda_n}\sinh(\lambda_n y)\right]$$

$$b_n=\frac{\int_0^L f_2(x)\sin(\lambda_n x)dx}{\left[\frac{L}{2}-\frac{\sin(2\lambda_n L)}{4\lambda_n}\right]\left[\lambda_n \sinh(\lambda_n M)-H_1\cosh(\lambda_n M)+H_2\left[\cosh(\lambda_n M-\frac{H_1}{\lambda_n}\sinh(\lambda_n M)\right]\right]}$$

$$u_3(x,y)=\sum_{n=1}^{\infty} c_n\left[\lambda_n \cos(\lambda_n y)+H_1\sin(\lambda_n y)\right]\left[\cosh(\lambda_n(x-L))-\frac{H_4}{\lambda_n}\sinh(\lambda_n(x-L))\right]$$

$$c_n=\frac{\int_0^M \left[\left[y(M-y)\right]\left[\lambda_n\cos(\lambda_n y)+H_1\sin(\lambda_n y)\right]\right]dy}{\frac{\left[\lambda_n^{2}+H_1^2\right]}{2}\left[L+\frac{H_2}{\left[\lambda_n^{2}+H_2^2\right]}+\frac{H_1}{2}\right]}$$

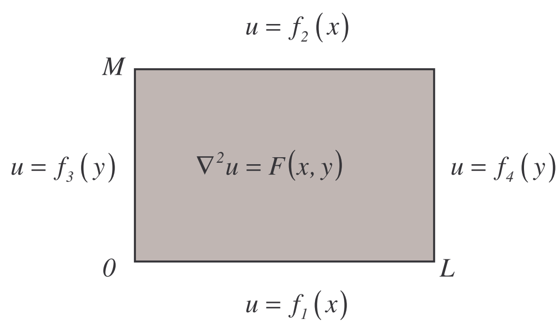
$$u_4(x,y)=\sum_{n=1}^{\infty} d_n\left[\lambda_n\cos(\lambda_n y)+H_1\sin(\lambda_n y)\right]\left[\sinh(\lambda_n x)\right]$$

$$d_n=\frac{\int_0^M f_4(y)\left[\lambda_n\cos(\lambda_n y)+H_1\sin(\lambda_n y)\right]dy}{\frac{\left[\lambda_n^{2}+H_1^2\right]}{2}\left[L+\frac{H_2}{\left[\lambda_n^{2}+H_2^2\right]}+\frac{H_1}{2}\right]\left[\lambda_n\cosh(\lambda_n L)+H_4\sinh(\lambda_n L)\right]}$$

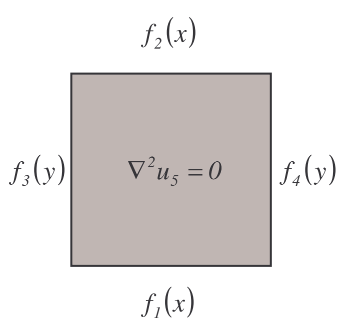
Solution of BVP problem:

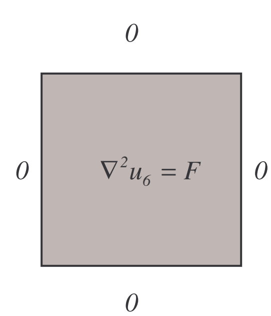
$$u(x,y)=u_1(x,y)+u_2(x,y)+u_3(x,y)+u_4(x,y)$$

**POISSON'S EQUATION: 01 – DDDD**




Supplemental problems: ↓




Solution of supplemental problems:

Solution of Laplace's homogeneous equation with non-homogeneous b.c.'s:

$$u_5(x, y) = u_1(x, y) + u_2(x, y) + u_3(x, y) + u_4(x, y)$$

Solution of Poisson's equation with homogeneous boundary conditions

$$u_6(x, y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{nm} \sin(\lambda_n x)[\mu_m \cos(\mu_m y) + H_1 \sin(\mu_m y)]$$

$$A_{nm} = \frac{\int_0^L \int_0^M F(x, y)[\sin(\lambda_n x)][\mu_m \cos(\mu_m y) + H_1 \sin(\mu_m y)]dydx}{\left[\frac{\mu_m^2 + H_1^2}{2}\left[L + \frac{H_2}{\mu_m^2 + H_2^2}\right] + \frac{H_1}{2}\right]\left[\frac{L}{2} - \frac{\sin(2\lambda_n L)}{4\lambda_n}\right][\lambda_n^2 + \mu_m^2]}$$

**<u>Solution of BVP for Poisson's Equation</u>** (superposition principle):

$$u(x, y) = u_5(x, y) + u_6(x, y)$$