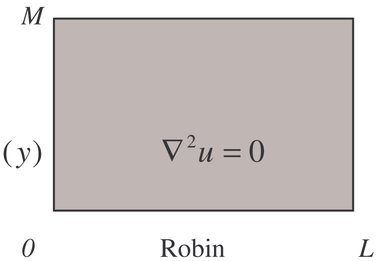
**The Laplace Equation: 01 – RRRR Robin- Robin - Robin - Robin)**

Robin

$$\left[-\frac{\partial u}{\partial y}+H_2 u\right]_{y=M} = f_2(x)$$

Robin

$$\left[-\frac{\partial u}{\partial x}+H_3 u\right]_{x=0} = f_3(y)$$



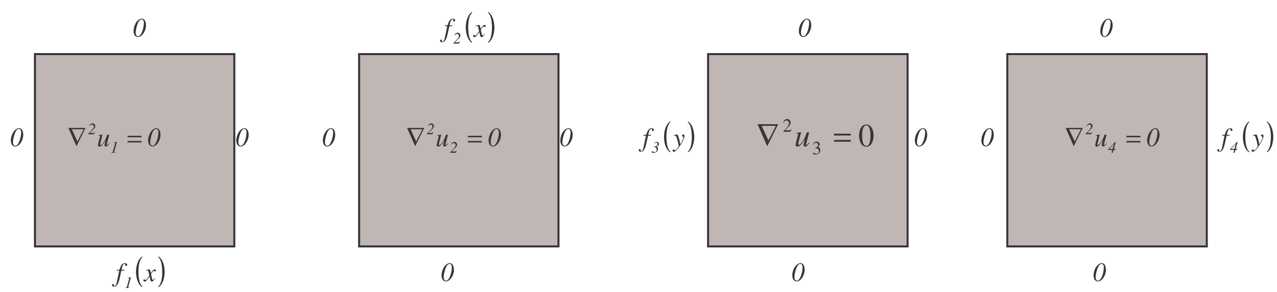

Robin

$$\left[-\frac{\partial u}{\partial x}+H_4 u\right]_{x=L} = f_4(y)$$

Robin

$$\left[-\frac{\partial u}{\partial y}+H_1 u\right]_{y=0} = f_1(x)$$

Supplemental problems: ↓




Solution of supplemental problems:

$$u_1(x,y)=\sum_{n=1}^{\infty} a_n (\cosh(\lambda_n(y-M)) - \frac{H_4}{\lambda_n}\sinh(\lambda_n(y-M)))(\lambda_n\cos(\lambda_n x)+H_3\sin(\lambda_n x))$$

$$a_n = \frac{\int_0^L f_1(x)(\lambda_n\cos(\lambda_n x)+H_3\sin(\lambda_n x)dx}{\left[\frac{1}{2}(\lambda_n^{\,2}+H_3^{\,2})(L+\frac{H_4}{(\lambda_n^{\,2}+H_4^{\,2})}+\frac{H_3}{2}\right]\left[\left(\lambda_n+\frac{H_3H_4}{\lambda_n}\right)\sinh\lambda_n M+(H_3+H_4)\cosh\lambda_n M\right]}$$

$$u_2(x,y)=\sum_{n=1}^{\infty} b_n (\cosh(\lambda_n(y)) + \frac{H_3}{\lambda_n}\sinh(\lambda_n(y)))(\lambda_n\cos(\lambda_n x)+H_3\sin(\lambda_n x))$$

$$b_n = \frac{\int_0^L f_2(x)(\lambda_n\cos(\lambda_n x)+H_3\sin(\lambda_n x)dx}{\left[\frac{1}{2}(\lambda_n^{\,2}+H_3^{\,2})(L+\frac{H_4}{(\lambda_n^{\,2}+H_4^{\,2})}+\frac{H_3}{2}\right]\left[\left(\lambda_n+\frac{H_3H_4}{\lambda_n}\right)\sinh\lambda_n M+(H_3+H_4)\cosh\lambda_n M\right]}$$

Where $\lambda_n$ are positive roots of: $(H_3H_4-\lambda_n^{\,2})\sin\lambda_n L+(H_3+H_4)\lambda_n\cos\lambda_n L=0$

$$u_3(x,y)=\sum_{n=1}^{\infty} c_n(\cosh(\lambda_n(x-L))-\frac{H_2}{\lambda_n}\sinh(\lambda_n(x-L)))(\lambda_n\cos(\lambda_n y)+H_1\sin(\lambda_n y))$$

$$c_n=\frac{\int_0^M f_3(y)(\lambda_n\cos(\lambda_n y)+H_1\sin(\lambda_n y)dy}{\left[\frac{1}{2}(\lambda_n^2+H_1^2)(L+\frac{H_2}{(\lambda_n^2+H_2^2)}+\frac{H_1}{2}\right]\left[\left(\lambda_n+\frac{H_1H_2}{\lambda_n}\right)\sinh\lambda_n M+(H_1+H_2)\cosh\lambda_n M\right]}$$

$$u_4(x,y)=\sum_{n=1}^{\infty} d_n\cosh(\lambda_n(x))+\frac{H_1}{\lambda_n}\sinh(\lambda_n(x)))(\lambda_n\cos(\lambda_n y)+H_1\sin(\lambda_n y))$$

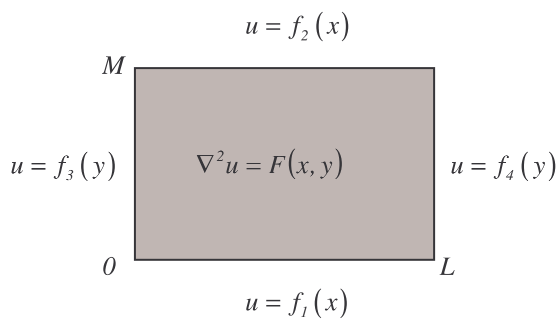
$$d_n=\frac{\int_0^M f_4(y)(\lambda_n\cos(\lambda_n y)+H_1\sin(\lambda_n y)dy}{\left[\frac{1}{2}(\lambda_n^2+H_1^2)(L+\frac{H_2}{(\lambda_n^2+H_2^2)}+\frac{H_1}{2}\right]\left[\left(\lambda_n+\frac{H_1H_2}{\lambda_n}\right)\sinh\lambda_n M+(H_1+H_2)\cosh\lambda_n M\right]}$$

Where $\lambda_n$ are positive roots of: $(H_1H_2-\lambda_n^2)\sin\lambda_n M+(H_1+H_2)\lambda_n\cos\lambda_n M=0$

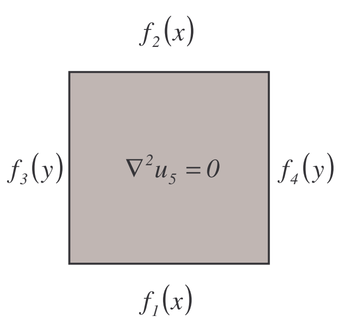
Solution of BVP problem:

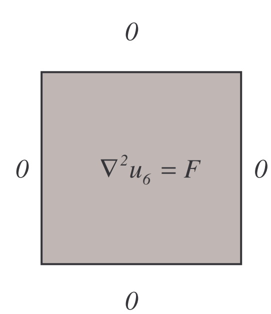
$$u(x,y)=u_1(x,y)+u_2(x,y)+u_3(x,y)+u_4(x,y)$$

**<u>POISSON'S EQUATION: 01 – RRRR</u>**




Supplemental problems: ↓



Solution of supplemental problems:

Solution of Laplace's homogeneous equation with non-homogeneous b.c.'s:

$$u_5(x,y) = u_1(x,y) + u_2(x,y) + u_3(x,y) + u_4(x,y)$$

Solution of Poisson's equation with homogeneous boundary conditions

$$u_6(x,y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{n,m}(\lambda_n \cos(\lambda_n x) + H_3 \sin(\lambda_n x))(\lambda_m \cos(\lambda_m y) + H_1 \sin(\lambda_m y))$$

$$A_{n,m} = \frac{\int_0^M\int_0^L F(x,y)(\lambda_n \cos(\lambda_n x) + H_3 \sin(\lambda_n x))(\lambda_m \cos(\lambda_m y) + H_1 \sin(\lambda_m y))dxdy}{\frac{1}{2}(\lambda_n{}^2 + \lambda_m{}^2)\left(\frac{1}{2}(\lambda_n{}^2 + H_3{}^2)\left(L + \frac{H_4}{\lambda_n{}^2 + H_4{}^2}\right) + \frac{H_3}{2}\right)\left(\frac{1}{2}(\lambda_m{}^2 + H_1{}^2)\left(M + \frac{H_2}{\lambda_m{}^2 + H_2{}^2}\right) + \frac{H_1}{2}\right)}$$

Where $\lambda_n$ are positive roots of: $(H_3 H_4 - \lambda_n{}^2)\sin\lambda_n L + (H_3 + H_4)\lambda_n \cos\lambda_n L = 0$

Where $\lambda_m$ are positive roots of: $(H_1 H_2 - \lambda_m{}^2)\sin\lambda_m M + (H_1 + H_2)\lambda_m \cos\lambda_m M = 0$

**Solution of BVP for Poisson's Equation** (superposition principle):

$$u(x,y) = u_5(x,y) + u_6(x,y)$$