fourier series

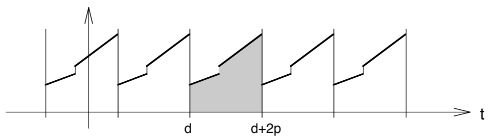
basic case $(d, d+2p)$

standard form

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi}{p}t + b_n \sin\frac{n\pi}{p}t\right]$$

complex exponential forms

$$= \sum_{n=-\infty}^{\infty} c_n e^{\frac{ni\pi}{p}t} = \sum_{n=-\infty}^{\infty} c_{-n} e^{\frac{-ni\pi}{p}t}$$



complex Fourier coefficients

$$c_n = \frac{1}{2p}\int_d^{d+2p} f(t)e^{\frac{-n\pi i}{p}t}dt \qquad c_{-n} = \frac{1}{2p}\int_d^{d+2p} f(t)e^{\frac{n\pi i}{p}t}dt$$

Fourier coefficients

$$a_0 = \frac{1}{p}\int_d^{d+2p} f(t)dt$$

$$a_n = \frac{1}{p}\int_d^{d+2p} f(t)\cos\frac{n\pi}{p}tdt$$

$$b_n = \frac{1}{p}\int_d^{d+2p} f(t)\sin\frac{n\pi}{p}tdt$$

relations

$$a_0 = 2c_0$$

$$a_n = c_n + c_{-n}$$

$$b_n = i(c_n - c_{-n})$$

$$c_0 = \frac{a_0}{2}$$

$$c_n = \frac{a_n - ib_n}{2}$$

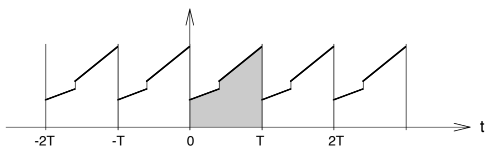
$$c_{-n} = \frac{a_n + ib_n}{2}$$

interval $(0, T)$

harmonic series

$$f(t) = c_0 + \sum_{n=1}^{\infty}\left[c_n \cos\frac{2\pi n}{T}t + d_n \sin\frac{2\pi n}{T}t\right] = \sum_{n=-\infty}^{\infty} A_n e^{\frac{i2\pi n}{T}t} = c_0 + \sum_{n=1}^{\infty}\sqrt{c_n^2 + d_n^2}\sin\left(\frac{2\pi n}{T}t + \phi_n\right)$$

$$\tan\phi_n = \frac{c_n}{d_n}$$



$$c_0 = \frac{1}{T}\int_0^T f(t)dt$$

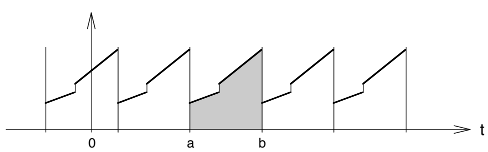
$$c_n = \frac{2}{T}\int_0^T f(t)\cos\frac{2\pi n}{T}tdt$$

$$d_n = \frac{2}{T}\int_0^T f(t)\sin\frac{2\pi n}{T}tdt$$

$$A_n = \frac{1}{T}\int_0^T f(t)e^{\frac{-2ni\pi}{T}t}dt$$

arbitrary interval (a, b)

$$f(t) = c_0 + \sum_{n=1}^{\infty}\left[c_n \cos\frac{2\pi n}{b-a}(t-a) + d_n \sin\frac{2\pi n}{b-a}(t-a)\right]$$



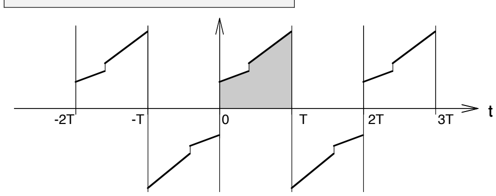
$$c_0 = \frac{1}{b-a}\int_a^b f(t)dt$$

$$c_n = \frac{2}{b-a}\int_a^b f(t)\cos\frac{2\pi n}{b-a}(t-a)dt$$

$$d_n = \frac{2}{b-a}\int_a^b f(t)\sin\frac{2\pi n}{b-a}(t-a)dt$$

sine series $(0, T)$

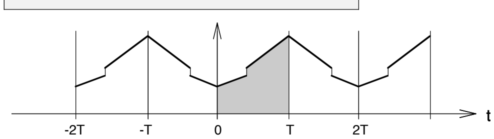
$$f(t) = \sum_{n=1}^{\infty} b_n \sin\frac{\pi n}{T}t$$



$$b_n = \frac{2}{T}\int_0^T f(t)\sin\frac{\pi n}{T}tdt$$

cosine series $(0, T)$

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{\pi n}{T}t$$

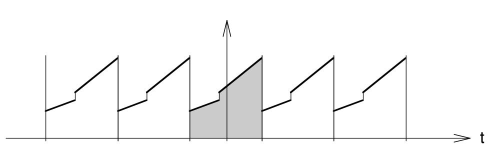


$$a_0 = \frac{1}{T}\int_0^T f(t)dt$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos\frac{\pi n}{T}tdt$$

symmetric interval $(-p, p)$

$$f(t) = \frac{c_0}{2} + \sum_{n=1}^{\infty}\left[c_n \cos\frac{\pi n}{p}t + d_n \sin\frac{\pi n}{p}t\right]$$



$$c_0 = \frac{1}{p}\int_{-p}^{p} f(t)dt$$

$$c_n = \frac{1}{p}\int_{-p}^{p} f(t)\cos\frac{\pi n}{p}tdt$$

$$d_n = \frac{1}{p}\int_{-p}^{p} f(t)\sin\frac{\pi n}{p}tdt$$