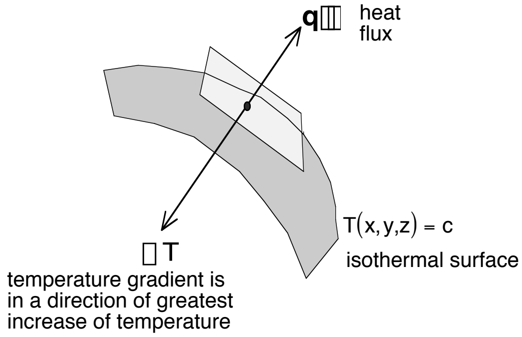
Fourier's law

heat conduction in continuous medium

$$\mathbf{q}'' = -k\nabla T$$

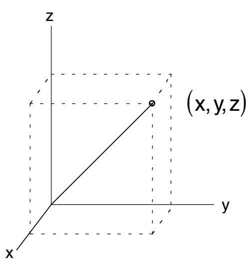


k	$\left[\frac{W}{m\cdot K}\right]$		coefficient of thermal conductivity
h	$\left[\frac{W}{m^2\cdot K}\right]$		coefficient of convective heat transfer
$\alpha = \frac{k}{\rho c_p}$	$\left[\frac{m^2}{s}\right]$		thermal diffusivity
$\dot{q}$	$\left[\frac{W}{m^3}\right]$		heat generation per unit volume
$q_g = \dot{q}V$	$[W]$		rate of heat generation
ε			surface emissivity

Heat Equation

Cartesian coordinates

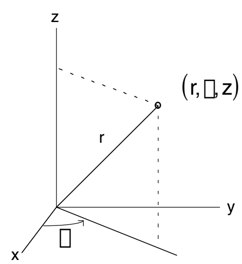
$$\mathbf{q}'' = \left\langle -k\frac{\partial T}{\partial x}, -k\frac{\partial T}{\partial y}, -k\frac{\partial T}{\partial z}\right\rangle$$



$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

cylindrical coordinates

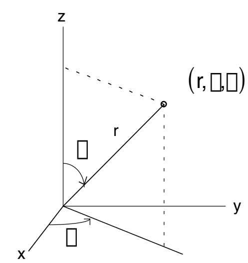
$$\mathbf{q}'' = \left\langle -k\frac{\partial T}{\partial r}, -\frac{k}{r}\frac{\partial T}{\partial \phi}, -k\frac{\partial T}{\partial z}\right\rangle$$



$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

spherical coordinates

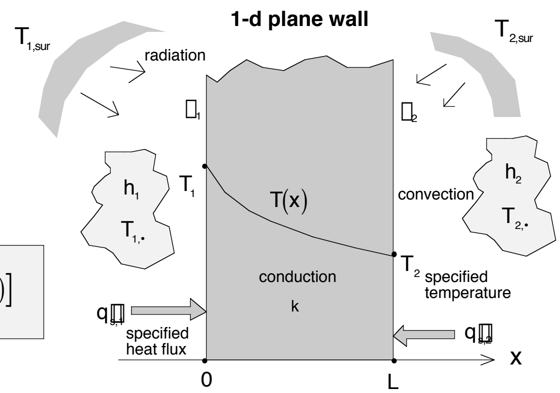
$$\mathbf{q}'' = \left\langle -k\frac{\partial T}{\partial r}, -\frac{k}{r\sin\theta}\frac{\partial T}{\partial \phi}, -\frac{k}{r}\frac{\partial T}{\partial \theta}\right\rangle$$



$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right) + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Boundary Conditions

1-d plane wall



non-linear boundary condtions:

at $x = 0$

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = h_1\left[T_{1,\infty} - T(0)\right] + \varepsilon_1\sigma\left[T^4_{1,sur} - T^4(0)\right]$$

at $x = L$

$$k\frac{\partial T}{\partial x}\bigg|_{x=L} = h_2\left[T_{2,\infty} - T(L)\right] + \varepsilon_2\sigma\left[T^4_{2,sur} - T^4(L)\right]$$

$\sigma = 5.67e-8\left[\frac{W}{m^2K^4}\right]$ Stefan-Boltzmann constant

classification of linearized boundary condtions:

I Dirichlet: $T\big|_{x=0} = T_1$ — constant surface temperature — $T\big|_{x=L} = T_2$

II Neumann: $-k\frac{\partial T}{\partial x}\bigg|_{x=0} = q''_{s1}$ — constant heat flux at the wall — $k\frac{\partial T}{\partial x}\bigg|_{x=L} = q''_{s2}$ — $\frac{\partial T}{\partial x}\bigg|_{x=0} = 0$ adiabatic surface: perfectly insulated surface (no flux thru the wall)

III Robin: $\left[-k\frac{\partial T}{\partial x} + h_1 T\right]_{x=0} = f_1$ — convective boundary condition — $\left[k\frac{\partial T}{\partial x} + h_2 T\right]_{x=L} = f_2$