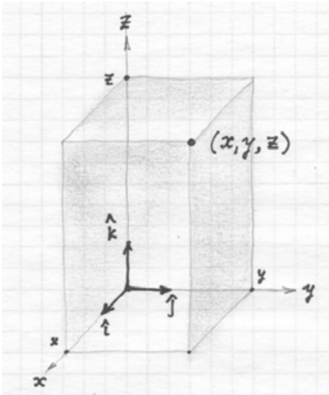
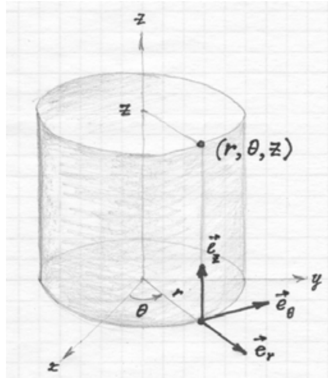
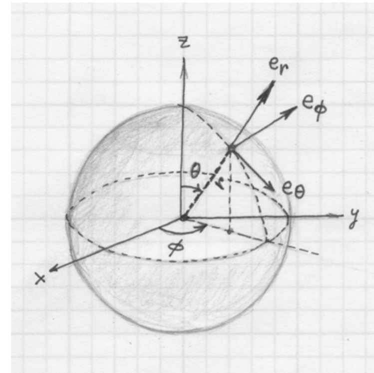
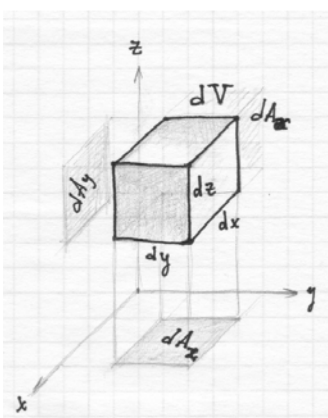
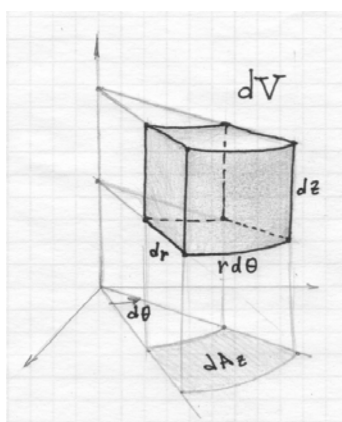
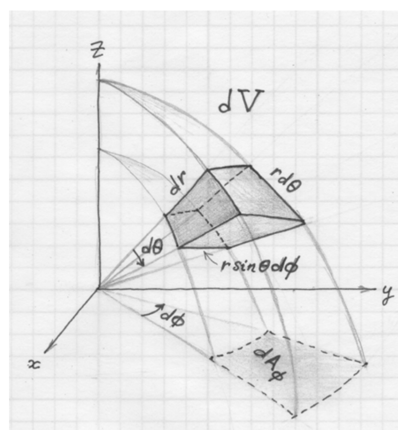
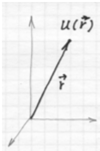
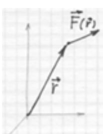
Coordinate Systems

Cartesian coordinates (x, y, z)	Cylindrical coordinates (r, θ, z)	Spherical coordinates (r, ϕ, θ)
	 $x = r\cos\theta$ $y = r\sin\theta$ $z = z$ $r^2 = x^2 + y^2$ $\tan\theta = \frac{y}{x}$ $z = z$	 $x = r\cos\phi\sin\theta$ $y = r\sin\phi\sin\theta$ $z = r\cos\theta$ $r^2 = x^2 + y^2 + z^2$ $\tan\phi = \frac{y}{x}$ $\tan\theta = \frac{z}{r} = \frac{z}{\sqrt{x^2+y^2+z^2}}$
Basic vectors $\mathbf{i} = (1,0,0)$ $\mathbf{j} = (0,1,0)$ $\mathbf{k} = (0,0,1)$	$\mathbf{e}_r = \mathbf{i}\cos\theta + \mathbf{j}\sin\theta$ $\mathbf{e}_\theta = -\mathbf{i}\sin\theta + \mathbf{j}\cos\theta$ $\mathbf{e}_z = \mathbf{k}$	$\mathbf{e}_r = \mathbf{i}\cos\phi\sin\theta + \mathbf{j}\sin\phi\sin\theta + \mathbf{k}\cos\theta$ $\mathbf{e}_\theta = -\mathbf{i}\sin\theta + \mathbf{j}\cos\phi$ $\mathbf{e}_\phi = \mathbf{i}\cos\phi\cos\theta + \mathbf{j}\sin\phi\cos\theta - \mathbf{k}\sin\theta$
 Line elements dx, dy, dz Differential areas $dA_x = dydz$ $dA_y = dxdz$ $dA_z = dxdy$ Differential volume $dV = dxdydz$ Arc length $ds^2 = dx^2 + dy^2 + dz^2$	 $dr, rd\theta, dz$ $dA_r = rd\theta dz$ $dA_\theta = drdz$ $dA_r = rd\theta dr$ $dV = rdrd\theta dz$ $ds^2 = dr^2 + r^2 d\theta^2 + dz^2$	 $dr,\ r\sin\theta d\phi,\ rd\theta$ $dA_r = r^2\sin\theta d\phi d\theta$ $dA_\phi = r\sin\theta d\phi dr$ $dA_\phi = \rho d\phi d\rho$ $dV = r^2\sin\theta d\phi d\theta dr$ $ds^2 = dr^2 + r^2\sin^2\theta d\phi + r^2 d\theta^2$

scalar field $u(\mathbf{r})$ 	$u(x,y,z)$	$u(r,\theta,z)$	$u(r,\phi,\theta)$
Gradient ∇u	$\nabla u = \left(\frac{\partial u}{\partial x},\frac{\partial u}{\partial y},\frac{\partial u}{\partial z}\right)$ $= \frac{\partial u}{\partial x}\mathbf{i}+\frac{\partial u}{\partial y}\mathbf{j}+\frac{\partial u}{\partial z}\mathbf{k}$	$\nabla u = \left(\frac{\partial u}{\partial r},\frac{1}{r}\frac{\partial u}{\partial \theta},\frac{\partial u}{\partial z}\right)$ $= \frac{\partial u}{\partial r}\mathbf{e}_r+\frac{1}{r}\frac{\partial u}{\partial \theta}\mathbf{e}_\theta+\frac{\partial u}{\partial z}\mathbf{e}_z$	$\nabla u = \left(\frac{\partial u}{\partial r},\frac{1}{r\sin\theta}\frac{\partial u}{\partial \phi},\frac{1}{r}\frac{\partial u}{\partial \theta}\right)$ $= \frac{\partial u}{\partial r}\mathbf{e}_r+\frac{1}{r\sin\theta}\frac{\partial u}{\partial \phi}\mathbf{e}_\phi+\frac{1}{r}\frac{\partial u}{\partial \theta}\mathbf{e}_\theta$
Laplacian $\nabla^2 u$	$\nabla^2 u = \frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}$	$\nabla^2 u = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+$ $+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}+\frac{\partial^2 u}{\partial z^2}$	$\nabla^2 u = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right)+$ $+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u}{\partial \phi^2}+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial u}{\partial \theta}\right)$
vector field $\mathbf{F}(\mathbf{r})$ 	(F_x,F_y,F_z)	(F_r,F_θ,F_z) $F_r = F_x\cos\theta+F_y\sin\theta$ $F_\theta = F_x\sin\theta+F_y\cos\theta$ $F_z = F_z$ $F_x = F_r\cos\theta-F_\theta\sin\theta$ $F_y = F_r\sin\theta+F_\theta\cos\theta$ $F_z = F_z$	(F_r,F_ϕ,F_θ) $F_r = F_x\cos\phi\sin\theta+F_y\sin\phi\sin\theta+F_z\cos\theta$ $F_\phi = -F_x\sin\phi+F_y\cos\phi$ $F_\theta = F_x\cos\phi\cos\theta+F_y\sin\phi\cos\theta-F_z\sin\theta$ $F_x = F_r\cos\phi\sin\theta-F_\phi\sin\phi+F_\theta\cos\phi\cos\theta$ $F_y = F_r\sin\phi\sin\theta+F_\phi\cos\phi+F_\theta\sin\phi\cos\theta$ $F_z = F_r\cos\theta-F_\theta\sin\theta$
Divergence $div\mathbf{F} = \nabla\cdot\mathbf{F}$	$\frac{\partial F_x}{\partial x}+\frac{\partial F_y}{\partial y}+\frac{\partial F_z}{\partial z}$	$\frac{1}{r}\frac{\partial}{\partial r}(rF_r)+\frac{1}{r}\frac{\partial F_\theta}{\partial \theta}+\frac{\partial F_z}{\partial z}$	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2F_r\right)+\frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial \phi}+$ $+\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left[(\sin\theta)F_\theta\right]$
$curl\mathbf{F} = \nabla\times\mathbf{F}$	$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ F_x & F_y & F_z\end{vmatrix} =$ $\left(\frac{\partial F_z}{\partial y}-\frac{\partial F_y}{\partial z}\right)\mathbf{i}+\left(\frac{\partial F_x}{\partial z}-\frac{\partial F_z}{\partial x}\right)\mathbf{j}+$ $+\left(\frac{\partial F_y}{\partial x}-\frac{\partial F_x}{\partial y}\right)\mathbf{k}$	$\frac{1}{r}\begin{vmatrix}\mathbf{e}_r & r\mathbf{e}_\theta & \mathbf{e}_z\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z}\\ F_r & rF_\theta & F_z\end{vmatrix} =$ $\left(\frac{1}{r}\frac{\partial F_z}{\partial \theta}-\frac{\partial F_\theta}{\partial z}\right)\mathbf{e}_r+$ $+\left(\frac{\partial F_r}{\partial z}-\frac{\partial F_z}{\partial r}\right)\mathbf{e}_\theta+$ $\frac{1}{r}\left[\frac{\partial(rF_\theta)}{\partial r}-\frac{\partial F_r}{\partial \theta}\right]\mathbf{e}_z$	$\frac{1}{r^2\sin\theta}\begin{vmatrix}\mathbf{e}_r & r\mathbf{e}_\theta & r\sin\theta\mathbf{e}_\phi\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi}\\ F_r & rF_\theta & r\sin\theta F_\phi\end{vmatrix} =$ $\frac{1}{r\sin\theta}\left[\frac{\partial\left(F_\phi\sin\theta\right)}{\partial \theta}-\frac{\partial F_\theta}{\partial \phi}\right]\mathbf{e}_r+$ $+\frac{1}{r}\left[\frac{\partial(rF_\theta)}{\partial r}-\frac{\partial F_r}{\partial \theta}\right]\mathbf{e}_\phi+$ $+\frac{1}{r\sin\theta}\left[\frac{\partial F_r}{\partial \phi}-\sin\theta\frac{\partial(rF_\phi)}{\partial r}\right]\mathbf{e}_\theta$