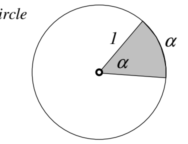
GEOMETRY OF RADIATION

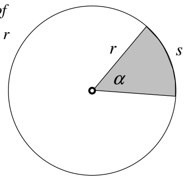
Plane Angle

unit circle



α = arc length [radian]

circle of radius r



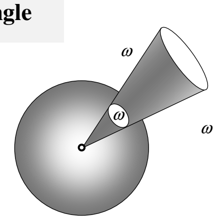
$$\frac{\alpha}{s} = \frac{2\pi \cdot 1}{2\pi \cdot r}$$

$$\Downarrow$$

$$\alpha = \frac{s}{r}$$

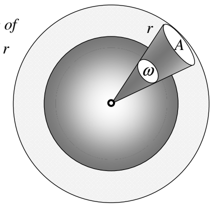
Solid Angle

unit sphere



ω = area on a unit sphere [steradian], [sr]

sphere of radius r

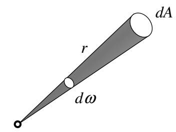


$$\frac{\omega}{A} = \frac{4\pi \cdot 1^2}{4\pi \cdot r^2}$$

$$\Downarrow$$

$$\omega = \frac{A}{r^2}$$

Differential Solid Angle



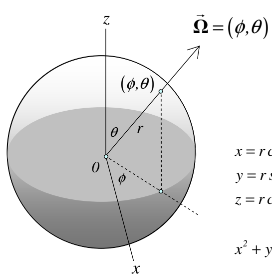
$$d\omega = \frac{dA}{r^2} \tag{12.7}$$

Differential Solid Angle in spherical coordinates

Direction is defined by a pair of angles: $\vec{\Omega} = (\phi, \theta)$

ϕ is an azimutal angle: $0 \le \phi \le 2\pi$

θ is a polar angle: $0 \le \theta \le \pi$

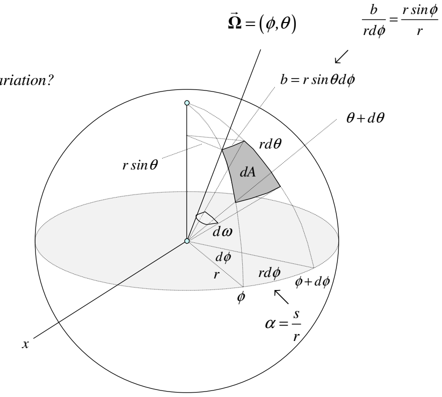


$$x = r\cos\phi\sin\theta$$
$$y = r\sin\phi\sin\theta$$
$$z = r\cos\theta$$

$$x^2 + y^2 + z^2 = r^2$$

Consider a differential variation of the direction $\vec{\Omega} = (\phi, \theta)$
$\phi + d\phi$ and $\theta + d\theta$

What solid anlgle corresponds to this variation?



$$dA \approx (r\sin\theta d\phi)\cdot(rd\theta) = r^2 \sin\theta d\phi d\theta$$

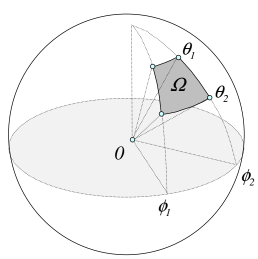
differential solid angle

$$d\omega = \frac{dA}{r^2} = \sin\theta d\theta d\phi \tag{12.8}$$

Finite Solid Angle in spherical coordinates

Consider a finite solid angle bounded by the directions:

$\phi_1 \le \phi \le \phi_2$ and $\theta_1 \le \theta \le \theta_2$



solid angle $$\Omega = \iint_{\Omega} d\omega = \int_{\phi_1}^{\phi_2}\int_{\theta_1}^{\theta_2} \sin\theta d\theta d\phi$$

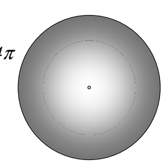
$$= -\int_{\phi=\phi_1}^{\phi_2}\int_{\mu=\cos(\theta_1)}^{\cos(\theta_2)} d\mu d\phi$$

substution $\mu = \cos\theta$

$$= (\phi_2 - \phi_1)(\cos\theta_1 - \cos\theta_2)$$

Sphere

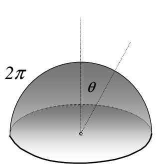
$\Omega_{sphere} = 4\pi$



$$\Omega_{sphere} = \Omega_{\bigcirc} = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \sin\theta d\theta d\phi = 4\pi$$

Hemisphere

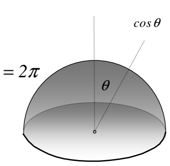
$\Omega_{hemisphere} = 2\pi$



$$\Omega_{hemisphere} = \int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi} \sin\theta d\theta d\phi = 2\pi$$

Useful Fact

$\Omega_{hemisphere} = 2\pi$



$$\int_{2\pi} \cos\theta d\omega = \int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi} \cos\theta \sin\theta d\theta d\phi = \pi$$