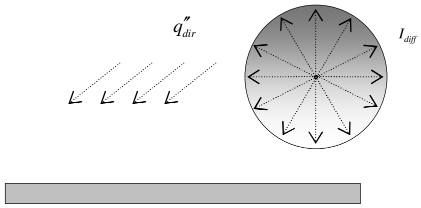
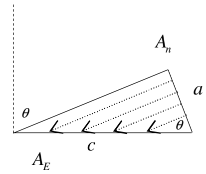
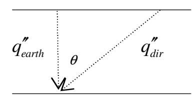
12.9





$$q''_{dir} = \frac{q_{incident}}{A_n} = 1000 \ \frac{W}{m^2}$$

$$q''_{earth\ surface,direct} = \left(\frac{q}{A_E} = q''_{dir} \cdot \frac{A_n}{A_E} = q''_{dir} \cdot \frac{a \cdot w}{c \cdot w} = q''_{dir} \cdot \frac{a}{c} \right) = q''_{dir} \cdot cos\,\theta$$



$$Solar\ irradiation = q''_{earth\ surface,direct} + q''_{earth\ surface,diffuse} = q''_{dir} \cdot cos\,\theta + \int_{2\pi} I_{diff} \cdot cos\,\theta d\omega$$

$$= q''_{dir} \cdot cos\,\theta + I_{diff} \cdot \int_{2\pi} cos\,\theta d\omega$$

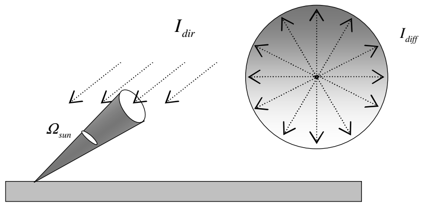
$$= q''_{dir} \cdot cos\,\theta + \pi I_{diff}$$

$$= (1000) \cdot \left(cos \frac{\pi}{6} \right) + \pi \cdot (70)$$

$$= 866 + 220$$

$$= 1086 \ \left[\frac{W}{m^2} \right]$$

12._10_



$$\Omega_{sun} = \frac{A_{cross}}{R^2} = \frac{\pi D_{sun}^2/4}{\left(R_{Earth-Sun} + R_{Sun}\right)^2} = \frac{\pi\left(1.39e9\right)^2 \cdot 0.25}{\left(1.496e11 + 1.39e9/2\right)^2} = 6.7e-5[sr]$$

$$Solar\ irradiation = q''_{earth\ surface,direct} + q''_{earth\ surface,diffuse}$$

$$= \int_{2\pi} I_{dir} \cdot \cos\theta d\omega + \int_{2\pi} I_{diff} \cdot \cos\theta d\omega$$

$$= \cos\theta_{incident} \cdot I_{dir} \cdot \int_{\Omega_{sun}} d\omega + I_{diff} \cdot \int_{2\pi} \cos\theta d\omega$$

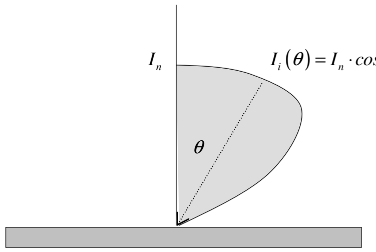
$$= \cos\theta_{incident} \cdot I_{dir} \cdot \left(\Omega_{sun}\right) + I_{diff} \cdot (\pi)$$

$$= \left(\cos\frac{\pi}{6}\right) \cdot (2.1e7) \cdot (6.78e-5) + (70) \cdot (\pi)$$

$$= 1233 + 220$$

$$= 1433 \left[\frac{W}{m^2}\right]$$

12.*11*



$$\text{Solar irradiation} = q''_{\text{earth surface,incident}}$$

$$= \int_{2\pi} I_i(\theta) \cdot \cos\theta d\omega$$

$$= I_n \cdot \int_{2\pi} \cos^2\theta d\omega$$

$$= I_n \cdot \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \cos^2\theta \sin\theta d\theta d\phi$$

integrate with respect to ϕ:
integrand does not depend on ϕ

$$= I_n \cdot (2\pi) \cdot \int_{\theta=0}^{\pi/2} \cos^2\theta \sin\theta d\theta$$

$$= -I_n \cdot (2\pi) \cdot \int_{\theta=0}^{\pi/2} \cos^2\theta d(\cos\theta)$$

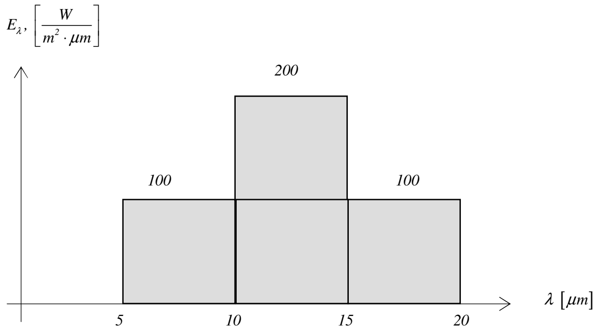
$$= -I_n \cdot (2\pi) \cdot \left[\frac{\cos^3\theta}{3} \right]_0^{\pi/2}$$

$$= -I_n \cdot (2\pi) \cdot \left[0 - \frac{1}{3} \right]$$

$$= \frac{I_n \cdot (2\pi)}{3}$$

$$= 167.5 \left[\frac{W}{m^2} \right]$$

12.1*6*



$$E = \int_0^\infty E_\lambda d\lambda = 5\cdot(100+200+100) = 2000 \left[\frac{W}{m^2}\right]$$

For diffuse surface:

$$I_e = \frac{E}{\pi} = \frac{2000}{\pi} = 636.6 \left[\frac{W}{m^2}\right]$$

$$\text{emissive power} = q''_{\text{emitted into the given solid angle}}$$

$$= \int_\Omega I_e \cdot \cos\theta d\omega$$

$$= I_e \cdot \int_{\phi=0}^{2\pi}\int_{\theta=\pi/4}^{\pi/2} \cos\theta \sin\theta d\theta d\phi$$

for integration
see Problem 12-9

$$= -I_e \cdot 2\pi \cdot \int_{\theta=\pi/4}^{\pi/2} \cos\theta d(\cos\theta)$$

$$= I_e \cdot 1.58$$

$$\text{Fraction} = \frac{I_e\cdot 1.58}{E} = \frac{I_e\cdot 1.58}{\pi I_e} = \frac{1.58}{\pi} = 0.5$$

12.17 $\textit{emissive power} = q''_{\text{emitted into the given solid angle}}$

$$= \int_{\Omega} I_e \cdot \cos\theta d\omega$$

$$= I_e \cdot \int_{\phi=0}^{\pi} \int_{\theta=\pi/4}^{\pi/2} \cos\theta \sin\theta d\theta d\phi$$

I_e is diffuse
(does not depend on θ)

$$= I_e \cdot \pi \cdot \int_{\theta=\pi/4}^{\pi/2} \cos\theta \sin\theta d\theta$$

$$= -I_e \cdot \pi \cdot \int_{\theta=\pi/4}^{\pi/2} \cos\theta d\left(\cos\theta\right)$$

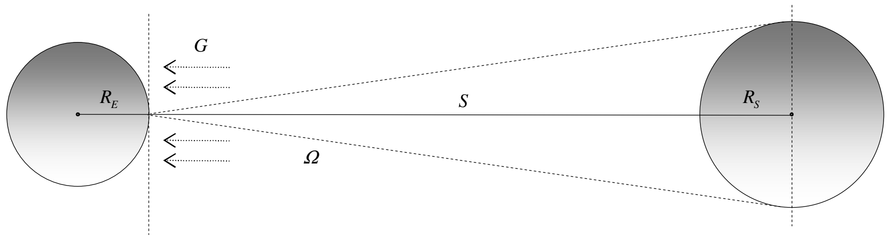
$$= -I_e \cdot \pi \cdot \left[\frac{\cos^2\theta}{2}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= -I_e \cdot \pi \cdot \left[\frac{\cos^2\frac{\pi}{2} - \cos^2\frac{\pi}{4}}{2}\right]$$

$$= -I_e \cdot \pi \cdot \left[\frac{0 - \cos^2\frac{\pi}{4}}{2}\right]$$

$$= I_e \cdot 0.79$$

$$\textit{Fraction} = \frac{I_e \cdot 0.79}{E} = \frac{I_e \cdot 0.79}{\pi I_e} = \frac{0.79}{\pi} = 0.25$$



$$G = 1368 \left[\frac{W}{m^2}\right]$$

$$\Omega = \frac{\pi R_S^2}{\left(S+R_S\right)^2} = 6.7e\text{-}5\left[sr\right]$$

Energy balance:

$$q = E_S \cdot A_S = E_S \cdot \left(4\pi R_S^2\right)$$

$$q = G \cdot A_{S+R_S} = G \cdot 4\pi\left(S+R_S\right)^2$$

$$\Rightarrow \quad E_S \cdot \left(4\pi R_S^2\right) = G \cdot 4\pi\left(S+R_S\right)^2 \quad \Rightarrow \quad E_S = G \cdot \frac{4\pi\left(S+R_S\right)^2}{\left(4\pi R_S^2\right)} = G \cdot \frac{\left(S+R_S\right)^2}{\left(R_S^2\right)}$$

$E = \pi I_e$ *total*

$G = \pi I_i$ *total*