Derivation of the Equations 13.0 - 13.1

Assumptions:

$$I_i = I_{emitted} + \rho I_{incident}$$

intensity leaving a surface is diffuse (does not depend on direction)
- it emmits diffusely
- any incident radiation is reflected diffusely
- surfaces are gray

non-participating medium between surfaces (vacuum, gas)
no effect on radiation exchange between surfaces

surfaces are gray and diffuse (emit, absorb and reflect diffusely)

radiation leaving the surface is considered to be
radiation leaving directly toward (means without intervening reflections)

irradiation is uniform

$$F_{dA_i \to dA_j} = \frac{intercepted}{leaving} = \frac{q_{dA_i \to dA_j}}{q_{dA_i}}$$

$$q_{dA_i} = (\pi I_i) \cdot dA_i$$

$$q_{dA_i \to dA_j} = I_i \cdot (dA_i \cdot cos\,\theta_i) \cdot d\omega_{i \to j} = I_i \cdot (dA_i \cdot cos\,\theta_i) \cdot \frac{dA_j \cdot cos\,\theta_j}{r^2}$$

$$d\omega_{i \to j} = \frac{A_{on\ sphere}}{r^2} = \frac{dA_j \cdot cos\,\theta_j}{r^2}$$

$$F_{dA_i \to dA_j} = \frac{\cancel{I_i} \cdot (\cancel{dA_i} \cdot cos\,\theta_i) \cdot \frac{dA_j \cdot cos\,\theta_j}{r^2}}{(\pi \cancel{I_i}) \cdot \cancel{dA_i}} = \frac{cos\,\theta_i \cdot cos\,\theta_j}{\pi r^2} dA_j$$

$$F_{dA_i \to dA_j} = \frac{cos\,\theta_i}{\pi} \cdot \frac{cos\,\theta_j \cdot dA_j}{r^2} \qquad (13.0)$$

$$F_{A_i \to A_j} = \frac{intercepted}{leaving} = \frac{q_{A_i \to A_j}}{q_{A_i}}$$

$$q_{A_i} = (\pi I_i) \cdot A_i$$

$$q_{A_i \to dA_j} = \int_{A_i} q_{dA_i \to dA_j} = \int_{A_i} \frac{dA_j \cdot cos\,\theta_j}{r^2} \cdot I_i \cdot (dA_i \cdot cos\,\theta_i)$$

$$q_{A_i \to A_j} = \int_{A_j} q_{A_i \to dA_j} = \int_{A_i}\int_{A_j} \frac{dA_j \cdot cos\,\theta_j}{r^2} \cdot I_i \cdot (dA_i \cdot cos\,\theta_i) = I_i \cdot \int_{A_i}\int_{A_j} \frac{cos\,\theta_i \cdot cos\,\theta_j}{r^2} dA_i dA_j$$

$$F_{A_i \to A_j} = \frac{q_{A_i \to A_j}}{q_{A_i}} = \frac{1}{(\pi \cancel{I_i}) \cdot A_i} \cdot \cancel{I_i} \cdot \int_{A_i}\int_{A_j} \frac{cos\,\theta_i \cdot cos\,\theta_j}{r^2} dA_i dA_j$$

$$F_{A_i \to A_j} = \frac{1}{A_i} \cdot \int_{A_i}\int_{A_j} \frac{cos\,\theta_i \cdot cos\,\theta_j}{\pi r^2}\, dA_i\, dA_j \qquad (13.1)$$

$$F_{dA_i \to A_j} = \frac{intercepted}{leaving} = \frac{q_{dA_i \to A_j}}{q_{dA_i}}$$

$$q_{dA_i \to A_j} = \int_{A_j} q_{dA_i \to dA_j} = \int_{A_j} \frac{I_i \cdot cos\,\theta_i \cdot cos\,\theta_j \cdot dA_i \cdot dA_j}{r^2}$$

$$F_{dA_i \to A_j} = \frac{q_{dA_i \to A_j}}{q_{dA_i}} = \frac{1}{(\pi I_i) \cdot dA_i} \cdot \int_{A_j} \frac{I_i \cdot cos\,\theta_i \cdot cos\,\theta_j \cdot dA_i \cdot dA_j}{r^2}$$

$$F_{dA_i \to A_j} = \frac{1}{(\pi \cancel{I_i}) \cdot \cancel{dA_i}} \cdot \int_{A_j} \frac{\cancel{I_i} \cdot cos\,\theta_i \cdot cos\,\theta_j \cdot \cancel{dA_i} \cdot dA_j}{r^2}$$

$$F_{dA_i \to A_j} = \int_{A_j} \frac{cos\,\theta_i \cdot cos\,\theta_j}{\pi r^2} \cdot dA_j \qquad (13.00)$$

$$F_{A_i \to dA_j} = \frac{intercepted}{leaving} = \frac{q_{A_i \to dA_j}}{q_{A_i}}$$

$$q_{A_i} = (\pi I_i) \cdot A_i$$

$$q_{A_i \to dA_j} = \int_{A_i} q_{dA_i \to dA_j} = \int_{A_i} \frac{dA_j \cdot cos\,\theta_j}{r^2} \cdot I_i \cdot (dA_i \cdot cos\,\theta_i)$$

$$F_{A_i \to dA_j} = \frac{q_{A_i \to dA_j}}{q_{A_i}} = \frac{\int_{A_i} \frac{dA_j \cdot cos\,\theta_j}{r^2} \cdot \cancel{I_i} \cdot (dA_i \cdot cos\,\theta_i)}{(\pi \cancel{I_i}) \cdot A_i}$$

$$F_{A_i \to dA_j} = \frac{1}{A_i} \cdot \int_{A_i} \frac{cos\,\theta_j \cdot cos\,\theta_i}{\pi r^2} dA_j dA_i \qquad (13.000)$$