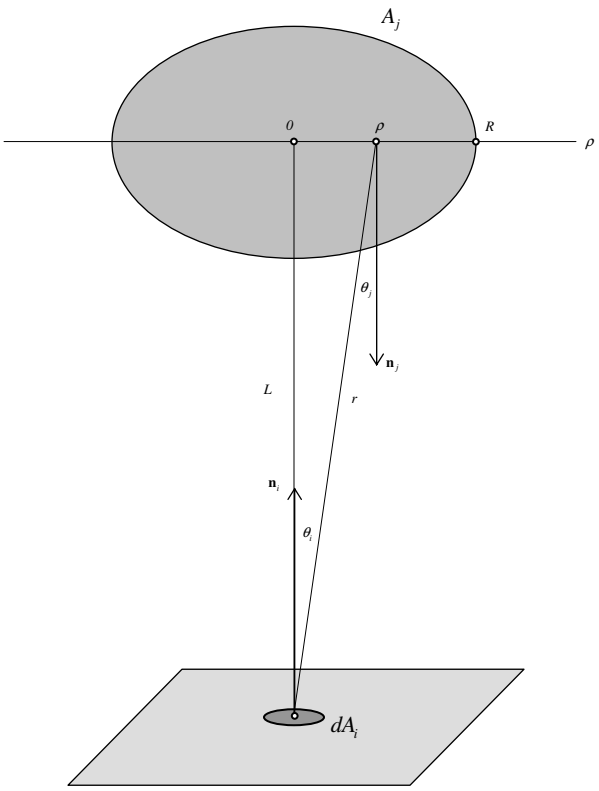
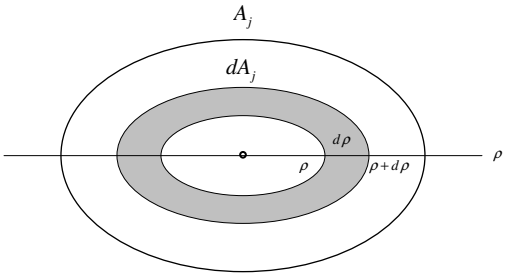
***Example 13.1 View factor of differential area to a disk***




$$dA_j = \pi(\rho + d\rho)^2 - \pi\rho^2 = \cancel{\pi\rho^2} + 2\pi\rho d\rho + \pi\overbrace{(d\rho)^2}^{<<d\rho} - \cancel{\pi\rho^2} = 2\pi\rho d\rho$$

$$\theta_i = \theta_j = \theta$$

$$r^2 = \rho^2 + L^2$$

$$\cos\theta = \frac{L}{r}$$

$$\cos^2\theta = \frac{L^2}{r^2} = \frac{L^2}{\rho^2 + L^2}$$

*areas $A_j$ and $dA_i$ are in the parallel planes*

*normal vectors $\mathbf{n}_i$ and $\mathbf{n}_j$ are parallel*

$$F_{dA_i \to A_j} = \int_{A_j} \frac{\cos\theta_i \cdot \cos\theta_j}{\pi r^2} \cdot dA_j \qquad (13.00)$$

*Find* $F_{dA_i \to A_j} = ?$

$$F_{dA_i \to A_j} = 2\int_{\rho=0}^{R} \frac{L^2}{r^2 + L^2} \frac{1}{\rho^2 + L^2}\, \rho d\rho$$

$$F_{dA_i \to A_j} = 2L^2 \int_{\rho=0}^{R} \frac{1}{\left[\rho^2 + L^2\right]^2}\, \rho d\rho$$

*u-substitution:*

$u = \rho^2 + L^2$

$du = 2\rho d\rho$

$$F_{dA_i \to A_j} = L^2 \int_{u=L^2}^{L^2+R^2} \frac{1}{u^2}\, du$$

$$F_{dA_i \to A_j} = L^2\left[-\frac{1}{u}\right]_{L^2}^{L^2+R^2} = -L^2\left[\frac{1}{L^2+R^2} - \frac{1}{L^2}\right] = \left[\frac{R^2}{L^2+R^2}\right] = \frac{R^2}{L^2+R^2}$$

$$\boxed{F_{dA_i \to A_j} = \frac{1}{1 + \left(\frac{L}{R}\right)^2}}$$