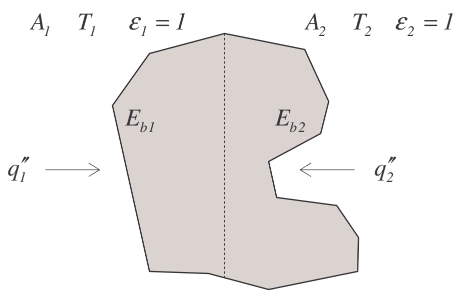
2-surface problem *(derivation of Eqn. 13.23)*:



matrix of view factors:

$$\begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}$$

$$F_{11} = 1 - F_{12}$$

$$F_{21} = \frac{A_1}{A_2} F_{12} \qquad \frac{F_{21}}{F_{12}} = \frac{A_1}{A_2}$$

$$F_{22} = 1 - \frac{A_1}{A_2} F_{12}$$

matrix of view factors:

$$\begin{bmatrix} 1 - F_{12} & F_{12} \\ \frac{A_1}{A_2} F_{12} & 1 - \frac{A_1}{A_2} F_{12} \end{bmatrix}$$

Given: $\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \Rightarrow \begin{bmatrix} E_{b1} = \sigma T_1^4 \\ E_{b2} = \sigma T_2^4 \end{bmatrix} \Rightarrow \begin{bmatrix} E_{b1} \\ E_{b2} \end{bmatrix}$ Find: $\begin{bmatrix} q_1'' \\ q_2'' \end{bmatrix}$

System of equations of NRM:

$$\frac{q_1''}{\varepsilon_1} - F_{11}\left(\frac{1}{\varepsilon_1} - 1\right) q_1'' - F_{12}\left(\frac{1}{\varepsilon_2} - 1\right) q_2'' = F_{12}\left(E_{b1} - E_{b2}\right)$$

$$\frac{q_2''}{\varepsilon_2} - F_{21}\left(\frac{1}{\varepsilon_1} - 1\right) q_1'' - F_{22}\left(\frac{1}{\varepsilon_2} - 1\right) q_2'' = F_{21}\left(E_{b2} - E_{b1}\right)$$

$$\left[\frac{1}{\varepsilon_1} - \left(1 - F_{12}\right)\left(\frac{1}{\varepsilon_1} - 1\right)\right] q_1'' - F_{12}\left(\frac{1}{\varepsilon_2} - 1\right) q_2'' = F_{12}\left(E_{b1} - E_{b2}\right)$$

$$-\frac{A_1}{A_2} F_{12}\left(\frac{1}{\varepsilon_1} - 1\right) q_1'' + \left[\frac{1}{\varepsilon_2} - \left(1 - \frac{A_1}{A_2} F_{12}\right)\left(\frac{1}{\varepsilon_2} - 1\right)\right] q_2'' = \frac{A_1}{A_2} F_{12}\left(E_{b2} - E_{b1}\right)$$

Solve the system for q_1'' using the Cramer Rule:

$$\Delta = \left[\left(\frac{1}{\varepsilon_1} - 1\right) F_{12} + 1\right]\left[1 + \left(\frac{1}{\varepsilon_2} - 1\right)\frac{A_1}{A_2} F_{12}\right] - F_{12}\left(\frac{1}{\varepsilon_2} - 1\right)\frac{A_1}{A_2} F_{12}\left(\frac{1}{\varepsilon_1} - 1\right)$$

$$= 1 + \left(\frac{1}{\varepsilon_2} - 1\right)\frac{A_1}{A_2} F_{12} + \left(\frac{1}{\varepsilon_1} - 1\right) F_{12}$$

$$\Delta_1 = \left[F_{12}\left(E_{b1} - E_{b2}\right)\right]\left[1 + \left(\frac{1}{\varepsilon_2} - 1\right)\frac{A_1}{A_2} F_{12}\right] + \left[\frac{A_1}{A_2} F_{12}\left(E_{b2} - E_{b1}\right)\right] F_{12}\left(\frac{1}{\varepsilon_2} - 1\right)$$

$$= \left[F_{12}\left(E_{b1} - E_{b2}\right)\right]\left[1 + \left(\frac{1}{\varepsilon_2} - 1\right)\frac{A_1}{A_2} F_{12} - \frac{A_1}{A_2} F_{12}\left(\frac{1}{\varepsilon_2} - 1\right)\right] = F_{12}\left(E_{b1} - E_{b2}\right)$$

$$q_1'' = \frac{\Delta_1}{\Delta} = \frac{F_{12}\left(E_{b1} - E_{b2}\right)}{1 + \left(\frac{1}{\varepsilon_2} - 1\right)\frac{A_1}{A_2} F_{12} + \left(\frac{1}{\varepsilon_1} - 1\right) F_{12}} = \frac{E_{b1} - E_{b2}}{\frac{1}{F_{12}} + \left(\frac{1}{\varepsilon_2} - 1\right)\frac{A_1}{A_2} + \left(\frac{1}{\varepsilon_1} - 1\right)}$$

$$q_1'' = \frac{E_{b1} - E_{b2}}{\left(\frac{1}{\varepsilon_1} - 1\right) + \frac{1}{F_{12}} + \left(\frac{1}{\varepsilon_2} - 1\right)\frac{A_1}{A_2}}$$

Equation 13.23