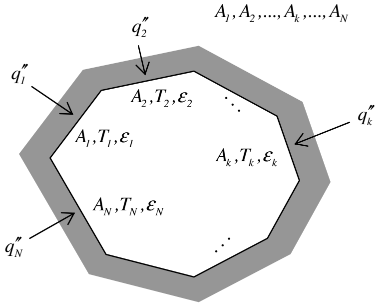
RADIATION EXCHANGE BETWEEN SURFACES

ENCLOSURE

enclosure consists of N surfaces $A_1, A_2, \ldots, A_k, \ldots, A_N$

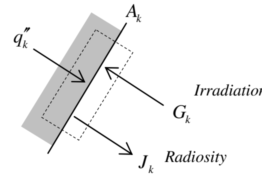


$$\sum_{k=1}^{N} q_k = 0$$ *energy balance*

$$q''_k = J_k - G_k \qquad (13.9)$$

$$q_k = q''_k \cdot A_k$$ *net rate of heat transfer to the surface* A_k

Net radiative flux at the surface A_k *has to be supplied* $(q''_k > 0)$ *or removed* $(q''_k < 0)$ *to maintain steady state*



radiosity $$J_k = (1-\varepsilon_k) G_k + \varepsilon_k E_{bk}$$

emissive power of a BB at T_k $$E_{bk} = \sigma T_k^4$$

Assumptions:

Each surface A_k *is*
- *isothermal at* T_k
- *diffuse*
- *gray with* $\varepsilon_k = \alpha_k$
- *with uniform radiosity* J_k
- *non-participating medium enclosed*
- *al view factors* $[F_{ij}]$ *are defined*
- *surfaces at the same temperature and emissivity can be considered as a single surface*
- *system is at steady state*

NET RADIATION METHOD

System of N equations, which include $q''_1, q''_2, \ldots, q''_N$ *and* $E_{b1}, E_{b2}, \ldots, E_{bN}$: *only N quantaties can be chosen as the unknowns:* (*either* q''_k *or* E_{bk} *for each of the surfaces*)

$$\frac{q''_k}{\varepsilon_k} - \left(\frac{1}{\varepsilon_1} - 1\right) F_{k1}\, q''_1 - \left(\frac{1}{\varepsilon_2} - 1\right) F_{k2}\, q''_2 - \ldots - \left(\frac{1}{\varepsilon_N} - 1\right) F_{kN}\, q''_N = F_{k1}\left(E_{bk} - E_{b1}\right) + \ldots + F_{kN}\left(E_{bk} - E_{bN}\right) \qquad \textbf{(NRM)} \qquad k = 1, 2, \ldots, N$$

Let be given: $\begin{bmatrix} q''_1 \\ T_2 \\ \vdots \\ T_N \end{bmatrix}$ *then emissive powers corresponding to given temperatures can be evaluated:* $E_{bk} = \sigma T_k^4$ $\Rightarrow$ $\begin{bmatrix} q''_1 \\ E_{b2} \\ \vdots \\ E_{bN} \end{bmatrix}$ $\Rightarrow$ *plug them into the NRM system*

Solve the **NRM** *for:* $\begin{bmatrix} E_{b1} \\ q''_2 \\ \vdots \\ q''_N \end{bmatrix}$ *then surface temperature corresponding to emmisive powers can be evaluated:* $T_k = (E_{bk}/\sigma)^{1/4}$ $\begin{bmatrix} T_1 \\ q''_2 \\ \vdots \\ q''_N \end{bmatrix}$ *then for each surface the net flux* q''_k *and temperature* T_k *is known* $\begin{bmatrix} T_1, & q''_1 \\ T_2, & q''_2 \\ & \vdots \\ T_N, & q''_N \end{bmatrix}$

BLACK SURFACES

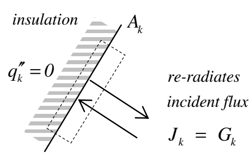
$$\varepsilon_1 = \varepsilon_2 = \ldots = \varepsilon_N = 1$$

$$q''_k = \sum_{j=1}^{N} F_{kj}\left(E_{bk} - E_{bj}\right) = \sigma \sum_{j=1}^{N} F_{kj}\left(T_k^4 - T_j^4\right) \qquad (13.17) \qquad k = 1, 2, \ldots, N$$

the NRM system is explicitely solved for net radiative fluxes q''_k

RE-RADIATING SURFACE

ADIABATIC (INSULATED) SURFACE

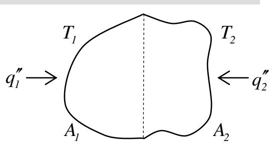


terms with q''_k *dissapear in the system* (**NRM**)

emissivity of insulated surface ε_k *does not influence radiation exchange*

temperature of insulated surface T_k *can be found from* (**NRM**)

2- SURFACE ECLOSURE

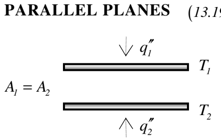


$$q_1 = q''_1 \cdot A_1$$
$$q_2 = q''_2 \cdot A_2 = -q''_1 \cdot A_1 \quad \Rightarrow \quad q_1 = -q_2, \qquad q''_2 = -q''_1 \cdot \frac{A_1}{A_2}$$

View Factors: $\begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}$ $\quad F_{11} = 1 - F_{12} \quad F_{21} = \frac{A_1}{A_2} F_{12} \quad F_{22} = 1 - \frac{A_1}{A_2} F_{21}$

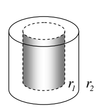
$$q''_1 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(\frac{1}{\varepsilon_1} - 1\right) + \frac{1}{F_{12}} + \left(\frac{1}{\varepsilon_2} - 1\right)\frac{A_1}{A_2}} \qquad (13.18)$$

PARALLEL PLANES (13.19)



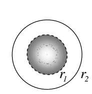
$$q''_1 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

CYLINDERS (13.20)



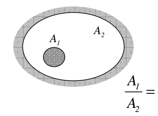
$$q''_1 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \left(\frac{1}{\varepsilon_2} - 1\right)\frac{r_1}{r_2}}$$

SPHERES (13.21)



$$q''_1 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \left(\frac{1}{\varepsilon_2} - 1\right)\left(\frac{r_1}{r_2}\right)^2}$$

LARGE CAVITY (SURROUNDINGS)

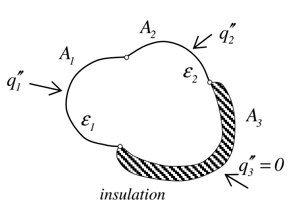


Stefan-Boltzmann Law:

$$q''_1 = \varepsilon_1 \sigma\left(T_1^4 - T_2^4\right)$$

3- SURFACE ECLOSURE

with one insulated (re-radiating) surface



$$q_1 + q_2 + \cancel{q_3} = 0 \quad \Rightarrow \quad q_1 = -q_2, \qquad q''_2 = -q''_1 \cdot \frac{A_1}{A_2}$$

$$q''_1 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(\frac{1}{\varepsilon_1} - 1\right) + \frac{1}{F_{12} + \left(\frac{1}{F_{13}} + \frac{1}{F_{23}}\frac{A_1}{A_2}\right)^{-1}} + \left(\frac{1}{\varepsilon_2} - 1\right)\frac{A_1}{A_2}} \qquad (13.25)$$

$k = 3$, $q''_3 = 0$, (**NRM**) $\Rightarrow$ $$-\left(\frac{1}{\varepsilon_1} - 1\right) F_{31}\, q''_1 - \left(\frac{1}{\varepsilon_2} - 1\right) F_{32}\, q''_2 = F_{31}\left(E_{b3} - E_{b1}\right) + F_{32}\left(E_{b3} - E_{b2}\right)$$ *solve for* E_{b3} *and then* $T_3 = \left(\frac{E_{b3}}{\sigma}\right)^{1/4}$