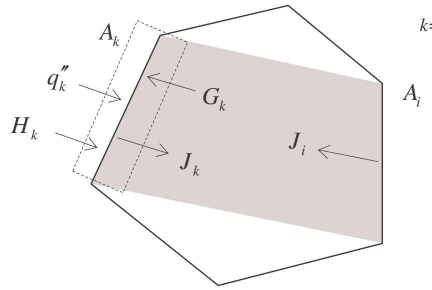
NET RADIATION METHOD
FOR ANALYSIS OF RADIATION EXCHANGE IN DIFFUSE-GRAY ENCLOSURES

Consider an N-surface enclosure:

H_k is imposed heat flux on the surface A_k (Modest calls it an external contribution to radiosity)

G_k is irradiation of the surface A_k by the radiosities from the surfaces of enclosure



$k=1,2,\ldots,N$ $\qquad \varepsilon_k = \alpha_k = 1-\rho_k$

Net radiative flux at the surface k:
$$q''_k + H_k = J_k - G_k \qquad (1)$$

Radiosity of the surface k:
$$J_k = (1-\varepsilon_k)G_k + \varepsilon_k E_{bk} \qquad (2)$$

Consider definition of the view factor:
$$F_{ik} = \frac{q_{i\to k}}{q_i} = \frac{q_{i\to k}}{J_i A_i} \quad \Rightarrow \quad q_{i\to k} = J_i A_i F_{ik}$$

Then irradiation of the surface k
$$G_k = \frac{1}{A_k}\sum_{i=1}^{N} q_{i\to k} = \frac{1}{A_k}\sum_{i=1}^{N} J_i A_i F_{ik} = \frac{1}{A_k}\sum_{i=1}^{N} J_i A_k F_{ki} = \sum_{i=1}^{N} J_i F_{ki} \qquad (3)$$

Substitute (3) into (1)
$$\boxed{q''_k + H_k = J_k - \sum_{i=1}^{N} J_i F_{ki}} \qquad (4)$$

Substitute (3) into (2)
$$J_k = (1-\varepsilon_k)\sum_{i=1}^{N} J_i F_{ki} + \varepsilon_k E_{bk} \qquad (5)$$

Solve (5) for $\sum_{i=1}^{N} J_i F_{ki}$
$$\sum_{i=1}^{N} J_i F_{ki} = \frac{J_k - \varepsilon_k E_{bk}}{1-\varepsilon_k}$$

Substitute it into (4)
$$q''_k = J_k - \frac{J_k - \varepsilon_k E_{bk}}{1-\varepsilon_k}$$

it can be written also for surface i:

Then solve it for J_k
$$\boxed{J_k = E_{bk} - \left(\frac{1}{\varepsilon_k}-1\right)q''_k} \qquad \boxed{J_i = E_{bi} - \left(\frac{1}{\varepsilon_i}-1\right)q''_i}$$

and substitute them into (4)
$$q''_k + H_k = E_{bk} - \left(\frac{1}{\varepsilon_k}-1\right)q''_k - \sum_{i=1}^{N}\left[E_{bi} - \left(\frac{1}{\varepsilon_i}-1\right)q''_i\right]F_{ki}$$

note that $E_{bk} = E_{bk}\underbrace{\sum_{i=1}^{N} F_{ki}}_{1} = \sum_{i=1}^{N} E_{bk} F_{ki}$
$$= E_{bk} - \left(\frac{1}{\varepsilon_k}-1\right)q''_k - \sum_{i=1}^{N} E_{bi}F_{ki} + \sum_{i=1}^{N}\left(\frac{1}{\varepsilon_i}-1\right)q''_i F_{ki}$$

$$= -\frac{q''_k}{\varepsilon_k} + q''_k + \sum_{i=1}^{N}(E_{bk}-E_{bi})F_{ki} + \sum_{i=1}^{N}\left(\frac{1}{\varepsilon_i}-1\right)q''_i F_{ki}$$

Then
$$\boxed{\frac{q''_k}{\varepsilon_k} - \sum_{i=1}^{N} F_{ki}\left(\frac{1}{\varepsilon_i}-1\right)q''_i + H_k = \sum_{i=1}^{N} F_{ki}(E_{bk}-E_{bi}) \qquad k=1,2,\ldots,N}$$

$$\boxed{\frac{q''_k}{\varepsilon_k} - F_{k1}\left(\frac{1}{\varepsilon_1}-1\right)q''_1 - F_{k2}\left(\frac{1}{\varepsilon_2}-1\right)q''_2 - \ldots - F_{kN}\left(\frac{1}{\varepsilon_N}-1\right)q''_N + H_k = F_{k1}(E_{bk}-E_{b1}) + F_{k2}(E_{bk}-E_{b2}) + \ldots + F_{kN}(E_{bk}-E_{bN})} \quad (\mathbf{NRM-H})$$