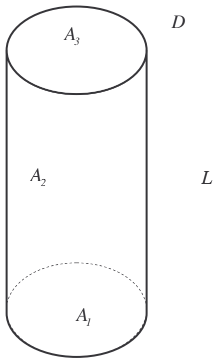
13.07



$$H = \frac{L}{D}$$

$$\begin{bmatrix} 0 & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & 0 \end{bmatrix}$$

a *Find* F_{12}

summation rule $\Rightarrow$ $F_{12} + F_{13} = 1 \Rightarrow F_{12} = 1 - F_{13}$

Table 13.2 $\Rightarrow$ $$S = 1 + \frac{1 + \left(\frac{D}{2L}\right)^2}{\left(\frac{D}{2L}\right)^2} = 1 + 1 + \left(\frac{2L}{D}\right)^2 = 2 + 4H^2$$

$$F_{13} = \frac{S - \left[S^2 - 4\left(\frac{R_3}{R_1}\right)^2\right]^{1/2}}{2} = \frac{S - \left[S^2 - 4\right]^{1/2}}{2}$$

$$= \frac{2 + 4H^2 - \left[4 + 16H^2 + 16H^4 - 4\right]^{1/2}}{2}$$

$$= \frac{2 + 4H^2 - \left[16H^2 + 16H^4\right]^{1/2}}{2}$$

$$= \frac{2 + 4H^2 - 4H\left[1 + H^2\right]^{1/2}}{2}$$

$$= 1 + 2H^2 - 2H\left[1 + H^2\right]^{1/2}$$

$$F_{12} = 1 - F_{13} \qquad = -2H^2 + 2H\left[1 + H^2\right]^{1/2} \qquad = 2H\left(\left[1 + H^2\right]^{1/2} - H\right)$$

b *Find* F_{22}

symmetry $\Rightarrow$ $F_{21} = F_{23}$

summation rule $\Rightarrow$ $F_{21} + F_{22} + F_{23} = 1 \Rightarrow 2F_{21} + F_{22} = 1 \Rightarrow F_{22} = 1 - 2F_{21} = 1 - 2\frac{A_1}{A_2}F_{12}$

$$= 1 - 2\frac{1}{4H}\left\{2H\left(\left[1 + H^2\right]^{1/2} - H\right)\right\}$$

$$\frac{A_2}{A_1} = \frac{\pi D \cdot L}{\frac{\pi D^2}{4}} = \frac{4L}{D} = 4H$$

$$= 1 - \left[1 + H^2\right]^{1/2} + H$$