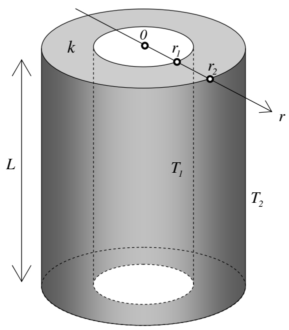
RADIAL SYSTEMS

1-D STEADY STATE CONDUCTION *without heat generation* $(\dot{q}=0)$

Cylindrical Wall



Heat Equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0$$

boundary conditions:

$$r=r_1 \quad T(r_1)=T_1$$

$$r=r_2 \quad T(r_2)=T_2$$

radial temperature:

$$T(r) = T_1-(T_1-T_2)\cdot\frac{ln\frac{r}{r_1}}{ln\frac{r_2}{r_1}}$$

temperature gradient:

$$\frac{\partial T}{\partial r} = \frac{T_1-T_2}{r\,ln\frac{r_1}{r_2}}$$

heat flux:

$$q'' = -k\frac{\partial T}{\partial r} = \frac{k}{r}\cdot\frac{T_1-T_2}{ln\frac{r_2}{r_1}}$$

heat rate:

$$q = q''A = 2\pi kL\cdot\frac{T_1-T_2}{ln\frac{r_2}{r_1}}$$

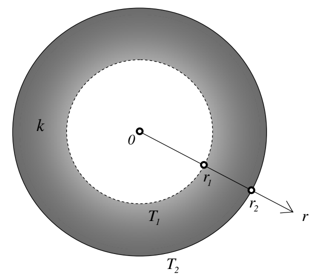
Thermal resistance

$$R_{cond} = \frac{T_1-T_2}{q} = \frac{ln\frac{r_2}{r_1}}{2\pi kL}$$

$$R_{conv} = \frac{1}{hA} = \frac{1}{h(2\pi rL)}$$

$$R_{rad} = \frac{1}{h_rA} = \frac{1}{h_r(2\pi rL)}$$

Spherical Wall



Heat Equation:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = 0$$

boundary conditions:

$$r=r_1 \quad T(r_1)=T_1$$

$$r=r_2 \quad T(r_2)=T_2$$

radial temperature:

$$T(r) = T_1-(T_1-T_2)\cdot\frac{1-\frac{r_1}{r}}{1-\frac{r_1}{r_2}}$$

temperature gradient:

$$\frac{\partial T}{\partial r} = \frac{1}{r^2}\cdot\frac{T_2-T_1}{\frac{1}{r_1}-\frac{1}{r_2}}$$

heat flux:

$$q'' = -k\frac{\partial T}{\partial r} = \frac{k}{r^2}\cdot\frac{T_1-T_2}{\frac{1}{r_1}-\frac{1}{r_2}}$$

heat rate:

$$q = q''A = 4\pi k\cdot\frac{T_1-T_2}{\frac{1}{r_1}-\frac{1}{r_2}}$$

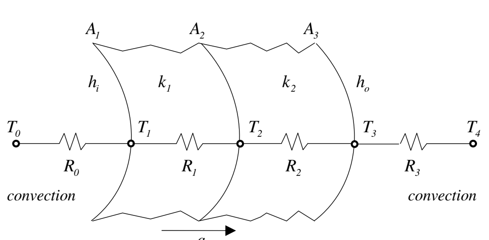
Thermal resistance

$$R_{cond} = \frac{T_1-T_2}{q} = \frac{\frac{1}{r_1}-\frac{1}{r_2}}{4\pi k}$$

$$R_{conv} = \frac{1}{hA} = \frac{1}{h(4\pi r^2)}$$

$$R_{rad} = \frac{1}{h_rA} = \frac{1}{h_r(4\pi r^2)}$$

Radial Composite System:



$$q = \frac{T_0-T_4}{R_{tot}} = U\cdot A\cdot(T_0-T_4)$$

$$R_{tot} = R_0+R_1+R_2+R_3$$

$$q = U_k\cdot A_k\cdot(T_1-T_4)$$

Where the overall heat transfer coefficient:

$$U = \frac{1}{A\cdot R_{tot}}$$

may be defined in terms of any area A_k :

$$U_k = \frac{1}{A_k\cdot R_{tot}}$$

$$U_1\cdot A_1 = U_2\cdot A_2 = \ldots = U_k\cdot A_k = \frac{1}{R_{tot}}$$