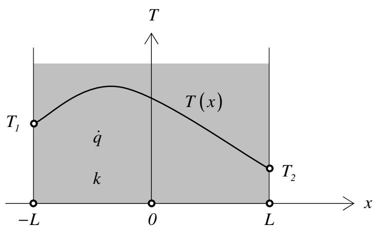
3.5 1-D STEADY STATE CONDUCTION *with uniform heat generation* $(\dot{q} = const)$

Plane Wall



Heat Equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = 0$$

boundary conditions:

$$x = -L \qquad T = T_1$$

$$x = L \qquad T = T_2$$

temperature profile:

$$T(x) = \frac{\dot{q}}{2k}\left(L^2 - x^2\right) + \frac{T_2 - T_1}{2L}x + \frac{T_1 + T_2}{2}$$

temperature gradient:

$$\frac{\partial T}{\partial x} = -\frac{\dot{q}}{k}x + \frac{T_2 - T_1}{2L}$$

heat flux:

$$q''(x) = -k\frac{\partial T}{\partial x} = \dot{q}x + k\frac{T_1 - T_2}{2L}$$

heat rate:

$$q(x) = q''(x)A = \left(\dot{q}x + k\frac{T_1 - T_2}{2L}\right)A$$

Symmetric case : $T_1 = T_2 = T_s$

temperature profile:

$$T(x) = \frac{\dot{q}}{2k}\left(L^2 - x^2\right) + T_s$$

middle point temperature: $T(0) = \frac{\dot{q}}{2k}L^2 + T_s$

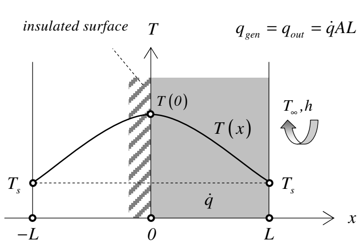
temperature gradient:

$$\frac{\partial T}{\partial x} = -\frac{\dot{q}}{k}x$$

$$\frac{\partial T}{\partial x}(0) = 0 \qquad \textit{insulated}$$

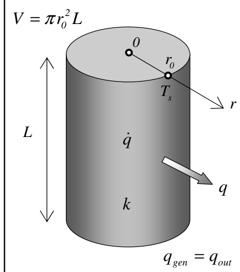
heat flux: $q''(x) = \dot{q}x$

heat rate: $q(x) = \dot{q}Ax$



Energy balance: $\dot{q}L = h(T_s - T_\infty)$ $\Rightarrow$ *Surface temperature:* $T_s = T_\infty + \frac{\dot{q}L}{h}$

Solid Cylinder



Heat Equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{q}}{k} = 0$$

boundary conditions:

$$r = r_0 \qquad T(r_0) = T_s$$

$$r = 0 \qquad T(0) < \infty$$

radial temperature:

$$T(r) = T_s + \frac{\dot{q}}{4k}\left(r_0^2 - r^2\right)$$

temperature gradient:

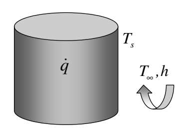
$$\frac{\partial T}{\partial r} = -\frac{\dot{q}r}{2k}$$

heat flux:

$$q'' = -k\frac{\partial T}{\partial r} = \frac{\dot{q}r}{2}$$

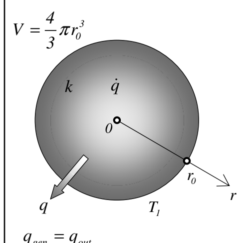
heat rate:

$$q = \dot{q}V = \left(\pi r^2 L\right)\dot{q}$$



Energy balance: $\dot{q}V = h(T_s - T_\infty)A$ $\Rightarrow$ *Surface temperature:* $T_s = T_\infty + \frac{\dot{q}r_0}{2h}$

Solid Sphere



Heat Equation:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{\dot{q}}{k} = 0$$

boundary conditions:

$$r = r_0 \qquad T(r_0) = T_s$$

$$r = 0 \qquad T(0) < \infty$$

radial temperature:

$$T(r) = T_s + \frac{\dot{q}}{6k}\left(r_0^2 - r^2\right)$$

temperature gradient:

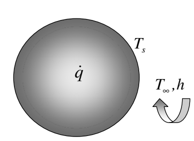
$$\frac{\partial T}{\partial r} = -\frac{\dot{q}r}{3k}$$

heat rate:

$$q = \dot{q}V = \frac{4}{3}\pi\dot{q}r_0^3$$

heat flux:

$$q'' = -k\frac{\partial T}{\partial r} = \frac{\dot{q}r}{3}$$



Energy balance: $\dot{q}V = h(T_s - T_\infty)A$ $\Rightarrow$ *Surface temperature:* $T_s = T_\infty + \frac{\dot{q}r_0}{3h}$