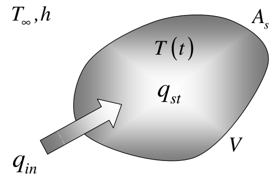
LUMPED CAPACITANCE METHOD

The temperature of the solid is assumed **spatially uniform**

The temperature is a function of time only $T(t)$



$$q_{in} = q_{st}$$

Energy Balance :

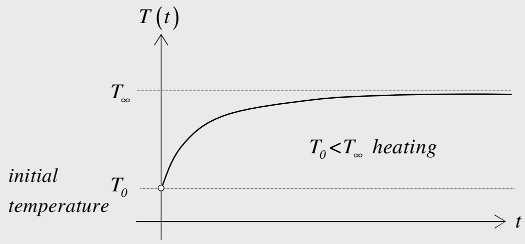
[net rate of heat transfer into the solid through its boundaries] = [rate of increase of the internal energy of the solid]

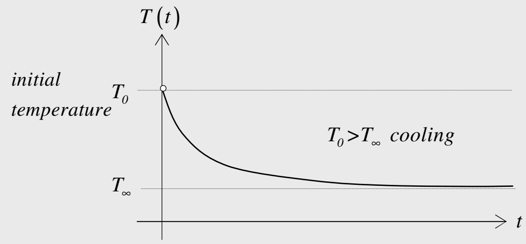
$$h(T_\infty - T)A_s = \rho c_p \frac{\partial T}{\partial t} V$$

Initial condition: $T(0) = T_0$

Temperature of a solid as a function of time

$$T(t) = T_\infty + (T_0 - T_\infty)e^{\frac{-h}{\rho c_p}\frac{A_s}{V}t} = T_\infty + (T_0 - T_\infty)e^{-\frac{t}{\tau}} = T_\infty + (T_0 - T_\infty)e^{-Bi \cdot Fo}$$





Biot Number: $Bi = \dfrac{hL_c}{k}$

Fourier Number: $Fo = \dfrac{\alpha t}{L_c^2}$

Time constant: $\tau = \dfrac{\rho c_p V}{hA_s}$

Biot Number

$$Bi = \frac{hL_c}{k} = \frac{\left(\frac{L_c}{kA_s}\right)}{\left(\frac{1}{hA_s}\right)} = \frac{\text{conduction thermal resistance}}{\text{convection thermal resistance}}$$

Lumped Capacitance is valid for

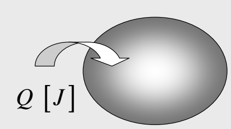
$$Bi < 0.1$$

Characteristic length

$$L_c = \frac{V}{A_s}$$

Total energy $Q\ [J]$ *transfered to a solid for time from 0 to t*

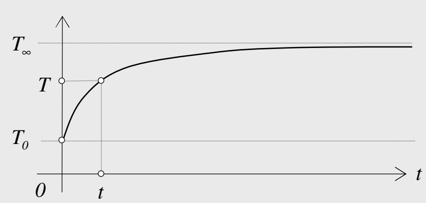
$$Q = \int_0^t q_{in}(t)\,dt = \int_0^t h\left[T_\infty - T(t)\right]A_s e^{\frac{-h}{\rho c_p}\frac{A_s}{V}t}\,dt = \rho c_p V(T_\infty - T_0)\left(1 - e^{\frac{-h}{\rho c_p}\frac{A_s}{V}t}\right)$$



$$Q = \rho c_p V(T_\infty - T_0) = mc_p \Delta T \qquad \text{when } t \to \infty$$

Time needed to heat a solid from T_0 *to current temperature* T

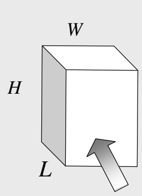
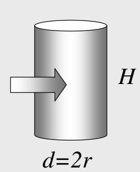
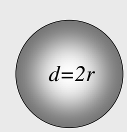
$$t = \frac{\rho c_p}{h}\frac{V}{A_s} ln\left(\frac{T_\infty - T_0}{T_\infty - T}\right)$$



Time Constant

$$\tau = \frac{\rho c_p V}{hA_s} = \underbrace{\rho c_p V}_{\text{thermal capacitance}} \cdot \underbrace{\frac{1}{hA_s}}_{\text{convection thermal resistance}}$$

The lower is the time constant τ *the faster is the heating of a solid*

Plane Wall	Cylinder	Sphere
		
$A_s = 2HW$	$A_s = \pi dH = 2\pi rH$	$A_s = \pi d^2 = 4\pi r^2$
$V = LHW$	$V = \frac{\pi d^2}{4}H = \pi r^2 H$	$V = \frac{\pi d^3}{6} = \frac{4}{3}\pi r^3$
$L_c = \frac{L}{2}$	$L_c = \frac{d}{4} = \frac{r}{2}$	$L_c = \frac{d}{6} = \frac{r}{3}$
$Bi = \frac{h}{k}\frac{L}{2}$	$Bi = \frac{h}{k}\frac{r}{2}$	$Bi = \frac{h}{k}\frac{r}{3}$