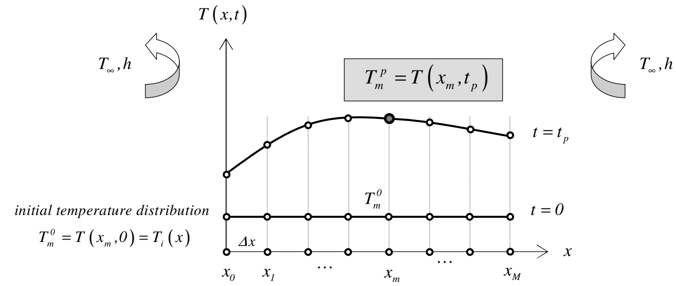
FINITE–DIFFERENCE METHOD – EXPLICIT METHOD

Heat Equation: $$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial x} \qquad 0 < x < L$$

Initial Condition: $$T(x,0) = T_i(x) \qquad 0 < x < L$$

Boundary Conditions:
$$-k\left.\frac{\partial T}{\partial x}\right|_{x=0} = h\left[T_\infty - T(0,t)\right] \qquad t > 0$$
$$-k\left.\frac{\partial T}{\partial x}\right|_{x=L} = h\left[T(L,t) - T_\infty\right] \qquad t > 0$$

$x_m = m \cdot \Delta x$	$m = 0,1,2,\ldots,M$	$\Delta x = \frac{L}{M}$	*step in space*
$t_p = p \cdot \Delta t$	$p = 0,1,2,\ldots$	Δt	*step in space*



The nodal network: *temperature field* $T_m^p = T(x_m, t_p)$ *will be determined only at the finite number of points (nodes)* x_m *and at discrete values of time* t_p

$m = 1,2,\ldots,M \qquad p = 0,1,2,\ldots$

Explicit Method:

$$Fo = \frac{\alpha \cdot \Delta t}{\Delta x^2} \qquad Bi = \frac{h \cdot \Delta x}{k} \qquad \alpha = \frac{k}{\rho c_p}$$

central-difference approximation:
$$\left.\frac{\partial^2 T}{\partial x^2}\right|_m^p \approx \frac{T_{m-1}^p - 2T_m^p + T_{m+1}^p}{\Delta x^2}$$

forward-difference approximation:
$$\left.\frac{\partial T}{\partial t}\right|_m^p \approx \frac{T_m^{p+1} - T_m^p}{\Delta t}$$

Interior Nodes (temperature at the next time level is calculated explicitly):

$$T_m^{p+1} = Fo\left(T_{m+1}^p + T_{m-1}^p\right) + (1-2Fo)T_m^p \qquad (5.78)$$

stable for: $$Fo \le \frac{1}{2} \qquad (5.79)$$

Boundary Nodes:

$$T_0^{p+1} = 2Fo\left(T_1^p + Bi \cdot T_\infty\right) + (1-2Fo-2BiFo)T_0^p \qquad (5.82)$$

$$T_M^{p+1} = 2Fo\left(T_{M-1}^p + Bi \cdot T_\infty\right) + (1-2Fo-2BiFo)T_M^p \qquad (5.82b)$$

stable for: $$Fo \cdot (1+Bi) \le \frac{1}{2} \qquad (5.84)$$

Solution Procedure:

- *Set initial temeperature distribution:* $T_m^0 = T(x_m,0) \quad m = 0,1,2,\ldots,M$

check for stability

- *Start the marching solution for* $p = 1,2,\ldots$:
  - *calculate at endpoints:* T_0^{p+1}, T_M^{p+1} *by* $(5.82, 5.82b)$
  - *calculate the rest of:* T_m^{p+1}, *m=1,2,...,M-1 by* (5.78)
  - *go to the next time level*