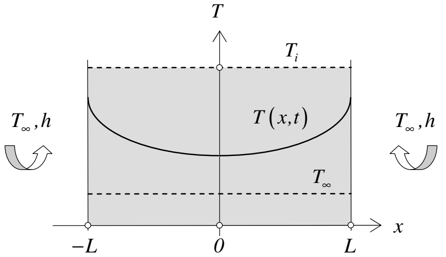
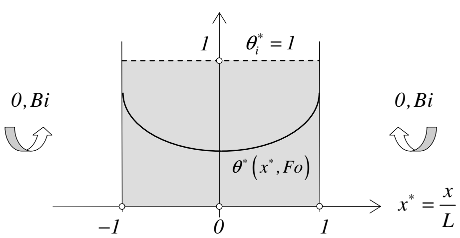
PLANE WALL



Heat Equation	$\dfrac{\partial^2 T}{\partial x^2} = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial t}$	$T(x,t): \quad x \in (0,L),\ t > 0$	
initial condition	$t = 0$	$T(x,0) = T_i$	(*constant*)
boundary conditions	$x = 0$	$\dfrac{\partial T}{\partial x} = 0$	(*symmetry*)
	$x = L$	$-k\dfrac{\partial T}{\partial x} = h(T - T_\infty)$	(*convective*)

NON – DIMENSIONALIZATION

$$Fo = \frac{\alpha t}{L^2} \qquad \theta^* = \frac{T - T_\infty}{T_i - T_\infty} \qquad Bi = \frac{hL}{k}$$



Heat Equation	$\dfrac{\partial^2 \theta^*}{\partial x^{*2}} = \dfrac{\partial \theta^*}{\partial Fo}$	$\theta^*(x^*, Fo): \quad x^* \in (0,1),\ Fo > 0$	
initial condition	$Fo = 0$	$\theta(x^*, 0) = 1$	(*constant*)
boundary conditions	$x^* = 0$	$\dfrac{\partial \theta}{\partial x^*} = 0$	(*symmetry*)
	$x^* = 1$	$\dfrac{\partial \theta^*}{\partial x^*} + Bi \cdot \theta^* = 0$	(*convective*)

Exact solution

$$\theta^* = \sum_{n=1}^{\infty} C_n \cos(\varsigma_n x^*)\, e^{-\varsigma_n^2 \cdot Fo} \qquad C_n = \frac{4 \sin \varsigma_n}{2\varsigma_n + \sin(2\varsigma_n)}$$

ς_n are positive roots of :

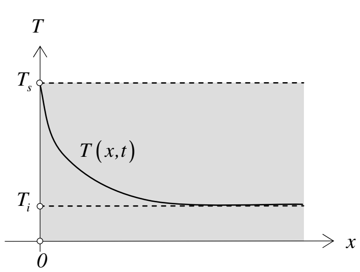
$$\varsigma \sin \varsigma - Bi \cdot \cos \varsigma = 0$$

Method of solution: separation of variables

5.7 SEMI – INFINITE SOLID

Heat Equation: $\dfrac{\partial^2 T}{\partial x^2} = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial t} \qquad T(x,t): \quad x > 0,\ t > 0 \qquad T(x,t) < \infty$

I



Boundary Condition: $T\big|_{x=0} = T_s$

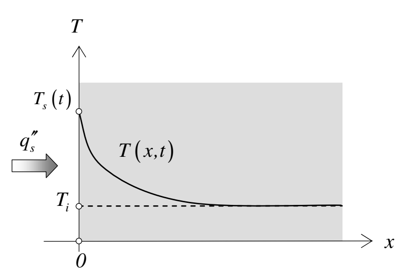
Exact Solution:

$$T(x,t) = T_s + (T_i - T_s) \cdot erf\left(\frac{x}{2\sqrt{\alpha t}}\right) \qquad (5.60)$$

Heat Flux:

$$q_s''(t) = \frac{k(T_s - T_i)}{\sqrt{\pi \alpha t}} \qquad (5.61)$$

II

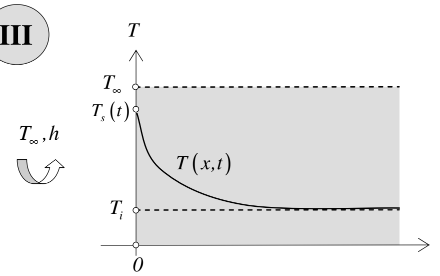


Boundary Condition: $-k\dfrac{\partial T}{\partial x}\bigg|_{x=0} = q_s''$

Exact Solution:

$$T(x,t) = T_i + \frac{2q_s''\sqrt{\alpha t/\pi}}{k} \cdot exp\left(\frac{-x^2}{4\alpha t}\right) - \frac{q_s'' x}{k} \cdot erfc\left(\frac{x}{2\sqrt{\alpha t}}\right) \qquad (5.62)$$

III



Boundary Condition: $-k\dfrac{\partial T}{\partial x}\bigg|_{x=0} = h\left(T_\infty - T\big|_{x=0}\right)$

Exact Solution:

$$\frac{T(x,t) - T_i}{T_\infty - T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right) - exp\left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right) \cdot erfc\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right) \qquad (5.63)$$

Method of solution: Laplace transform *Error function:* $y = erf(x) = \int_0^x e^{-s^2} ds$ *Complimentary error function:* $erfc(x) = 1 - erf(x)$ *Tabulated in B-1 , p.1015*