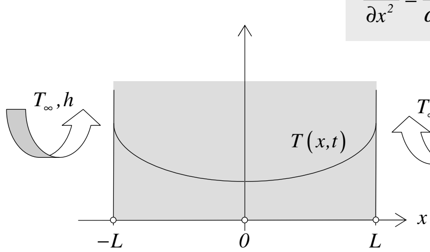
TRANSIENT CONDUCTION – APPROXIMATE ANALYTICAL SOLUTIONS

Approximation is valid for $Fo > 0.2$

initial temperature T_i *ambient temperature* T_∞ *non-dimensional temperature* $\Theta^* = \dfrac{T - T_\infty}{T_i - T_\infty}$ *non-dimensional coordinates* $x^* = \dfrac{x}{L}$ $r^* = \dfrac{r}{r_0}$ *non-dimensional time* $Fo = \dfrac{\alpha t}{L^2}$ $Fo = \dfrac{\alpha t}{r_0^2}$

PLANE WALL

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$



$$Fo = \frac{\alpha t}{L^2}$$

$$V = 2LWH$$

$$Bi = \frac{hL}{k} \xrightarrow{\text{Table 5.1 p. 301}} \varsigma_1, C_1$$

$$\Theta^* = C_1 \cos\left(\varsigma_1 x^*\right) e^{-\varsigma_1^2 \cdot Fo}$$

approximate solution (one term solution)

$$T(x,t) = T_\infty + (T_i - T_\infty) C_1 \cos\left(\varsigma_1 \frac{x}{L}\right) e^{-\varsigma_1^2 \frac{\alpha}{L^2} t}$$

approximate solution is valid for $Fo > 0.2$ *or* $t > 0.2 \dfrac{L^2}{\alpha}$

$$t = \frac{L^2}{\alpha \varsigma_1^2} \cdot ln\left[\frac{T_\infty - T_i}{T_\infty - T(x,t)} \cdot C_1 \cdot cos\left(\varsigma_1 \frac{x}{L}\right)\right] \quad [s]$$

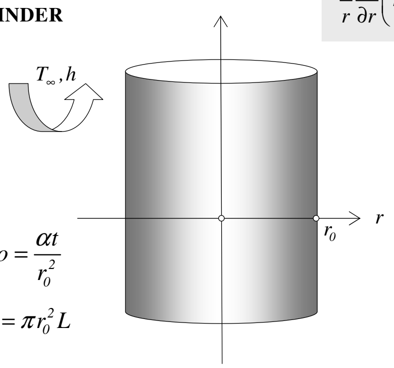
time needed to heat the wall at location x from T_i *to a current temperature* $T(x,t)$

$$Q = \left[1 - C_1 \frac{sin(\varsigma_1)}{\varsigma_1} e^{-\varsigma_1^2 \frac{\alpha}{L^2} t}\right] \cdot \rho c_p V \cdot (T_i - T_\infty) \quad [J]$$

total heat transferred from the wall over the time t

CYLINDER

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$



$$Fo = \frac{\alpha t}{r_0^2}$$

$$V = \pi r_0^2 L$$

$$Bi = \frac{h r_0}{k} \xrightarrow{\text{Table 5.1}} \varsigma_1, C_1$$

$$\Theta^* = C_1 J_0\left(\varsigma_1 r^*\right) e^{-\varsigma_1^2 \cdot Fo}$$

approximate solution

$J_0(x)$ *is a Bessel Function* $(B.4\ p.1017)$

$$T(r,t) = T_\infty + (T_i - T_\infty) \cdot C_1 \cdot J_0\left(\varsigma_1 \frac{r}{r_0}\right) \cdot e^{-\varsigma_1^2 \frac{\alpha}{r_0^2} t}$$

$$t = \frac{r_0^2}{\alpha \varsigma_1^2} \cdot ln\left[\frac{T_\infty - T_i}{T_\infty - T(r,t)} \cdot C_1 \cdot J_0\left(\varsigma_1 \frac{r}{r_0}\right)\right]$$

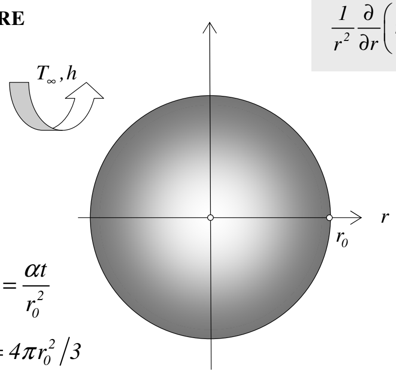
time needed to heat the cylinder at location r from T_i *to a current temperature* $T(r,t)$

$$Q = \left[1 - 2 C_1 J_1(\varsigma_1) e^{-\varsigma_1^2 \frac{\alpha}{L^2} t}\right] \cdot \rho c_p V \cdot (T_i - T_\infty)$$

total heat transferred from the cylinder over the time t

SPHERE

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$



$$Fo = \frac{\alpha t}{r_0^2}$$

$$V = 4\pi r_0^2 / 3$$

$$Bi = \frac{h r_0}{k} \xrightarrow{\text{Table 5.1}} \varsigma_1, C_1$$

$$\Theta^* = C_1 \frac{sin\left(\varsigma_1 r^*\right)}{\varsigma_1 r^*} e^{-\varsigma_1^2 \cdot Fo}$$

approximate solution

$$T(r,t) = T_\infty + (T_i - T_\infty) \cdot C_1 \cdot \frac{sin\left(\varsigma_1 \frac{r}{r_0}\right)}{\varsigma_1 \frac{r}{r_0}} \cdot e^{-\varsigma_1^2 \frac{\alpha}{r_0^2} t}$$

Note that at r=0 $\dfrac{sin\left(\varsigma_1 \frac{r}{r_0}\right)}{\varsigma_1 \frac{r}{r_0}} = 1$

$$t = \frac{r_0^2}{\alpha \varsigma_1^2} \cdot ln\left[\frac{T_\infty - T_i}{T_\infty - T(r,t)} \cdot \frac{C_1 \cdot sin\left(\varsigma_1 \frac{r}{r_0}\right)}{\varsigma_1 \frac{r}{r_0}}\right]$$

time needed to heat the sphere at location r from T_i *to a current temperature* $T(r,t)$

$$Q = \left[1 - \frac{3 C_1 \cdot (sin\,\varsigma_1 - \varsigma_1 \cdot cos\,\varsigma_1)}{\varsigma_1^3} \cdot e^{-\varsigma_1^2 \frac{\alpha}{r_0^2} t}\right] \cdot \rho c_p V \cdot (T_i - T_\infty)$$

total heat transferred from the sphere over the time t