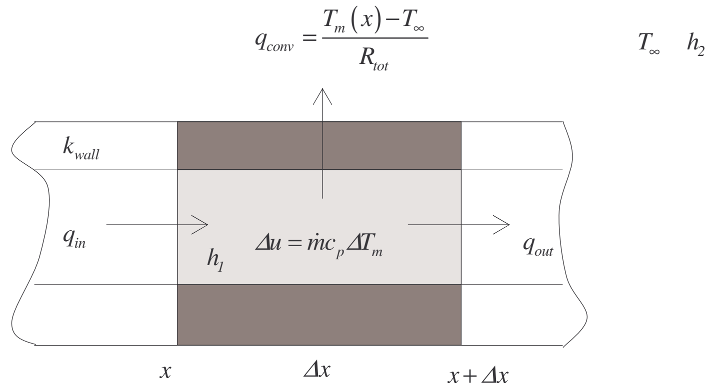
Derivation of the equation (8.45):

Energy balance for c.v.:



$$q_{out} - q_{in} = \dot{m} c_p \left[T_m(x+\Delta x) - T_m(x) \right] = \dot{m} c_p \Delta T_m = -\frac{T_m(x) - T_\infty}{R_{tot}}$$

Thermal resistance:

$$R_{tot} = R_{1,conv} + R_{wall} + R_{2,conv}$$

$$= \frac{1}{h_1 \left(\underbrace{2\pi r_1 \Delta x}_{\Delta A_1} \right)} + \frac{\ln \frac{r_2}{r_1}}{k_w (2\pi \Delta x)} + \frac{1}{h_2 \left(\underbrace{2\pi r_2 \Delta x}_{\Delta A_1} \right)}$$

$$= \frac{1}{\underbrace{2\pi r_1}_{P_1} \Delta x} \underbrace{\left[\frac{1}{h_1} + \frac{r_1}{k_w} \ln \frac{r_2}{r_1} + \frac{r_1}{r_2} \frac{1}{h_2} \right]}_{\text{denote as } U_1}$$

$$= \frac{1}{P_1 U_1 \Delta x}$$

$$\frac{1}{U_1(x)} = \frac{1}{h_1} + \frac{r_1}{k_w} \ln \frac{r_2}{r_1} + \frac{r_1}{r_2} \frac{1}{h_2}$$

$$\dot{m} c_p \Delta T_m = -P_1 U_1 \Delta x (T_m - T_\infty)$$

$$\dot{m} c_p \frac{\Delta T_m}{\Delta x} = -P_1 U_1 (T_m - T_\infty)$$

$$\frac{\Delta T_m}{\Delta x} = -\frac{P_1 U_1}{\dot{m} c_p} (T_m - T_\infty) \qquad \Delta x \to 0$$

$$\frac{dT_m}{dx} = -\frac{P_1 U_1}{\dot{m} c_p} (T_m - T_\infty) \qquad T_m(0) = T_i$$

$$\frac{d(T_m - T_\infty)}{dx} + \frac{P_1 U_1(x)}{\dot{m} c_p} (T_m - T_\infty) = 0 \qquad T_m(0) - T_\infty = T_i - T_\infty$$

Integrating factor (see Table with Linear ODE):

$$\mu = e^{\int_0^x \frac{P_1 U_1(x)}{\dot{m} c_p} dx} = e^{\frac{P_1}{\dot{m} c_p} \int_0^x U_1(x) dx} = e^{\frac{P_1 x}{\dot{m} c_p} \frac{\int_0^x U_1(x) dx}{x}} = e^{\frac{P_1 x}{\dot{m} c_p} \bar{U}_1(x)}$$

$$T_m(x) = T_\infty + (T_i - T_\infty) e^{-\frac{P_1 \bar{U}_1(x)}{\dot{m} c_p} x}$$

mean temperature profile

$$T_o = T_\infty + (T_i - T_\infty) e^{-\frac{P_1 \bar{U}_1(L)}{\dot{m} c_p} L}$$

outlet temperature

(This has to be inspected):

$$\frac{1}{\bar{U}_1(x)} = \frac{1}{\bar{h}_1} + \frac{r_1}{k_w} \ln \frac{r_2}{r_1} + \frac{r_1}{r_2} \frac{1}{\bar{h}_2}$$

$\bar{U}_1(x)$ is avaraged if $\bar{h}_1$ and $\bar{h}_2$ are averaged