9.1–4 **FREE CONVECTION – PHYSICAL CONSIDERATIONS**

Volumetric thermal expansion coefficient

$$\beta = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p , \quad \left[\frac{1}{K}\right]$$

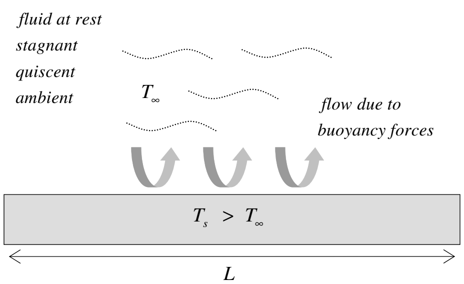
for ideal gas

$$\beta = \frac{1}{T} \quad \left[\frac{1}{K}\right]$$

properies at $T_f = \dfrac{T_s + T_\infty}{2}$ | $g = 9.8\left[\dfrac{m}{s^2}\right]$ | $\nu = \dfrac{\mu}{\rho}$

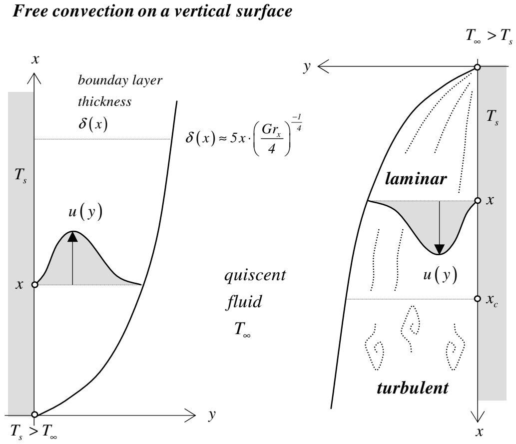
Grashof number

$$Gr_L = \frac{g\beta\left|T_s - T_\infty\right| L^3}{\nu^2} = \frac{\text{bouyancy force}}{\text{viscous force}} \qquad (9.12)$$



$Gr_L >> Re_L^2$	free convection	$Nu_L = f\left(Gr_L, Pr\right)$
$Gr_L \approx Re_L^2$	combined convection	$Nu_L = f\left(Re_L, Gr_L, Pr\right)$
$Gr_L << Re_L^2$	forced convection	$Nu_L = f\left(Re_L, Pr\right)$

Free convection on a vertical surface



Rayleigh number

$$Ra_L = Gr_L Pr = \frac{g\beta\left(T_s - T_\infty\right)L^3}{\alpha\nu} \qquad (9.25)$$

Critical Rayleigh number

$$Ra_{x,c} = 10^9$$

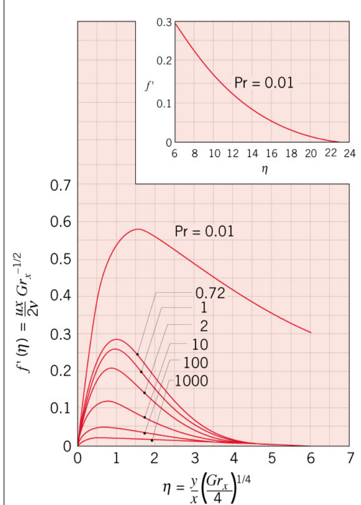
$$Ra_x = Gr_x Pr = \frac{g\beta\left(T_s - T_\infty\right)x^3}{\alpha\nu} \qquad (9.23)$$

velocity

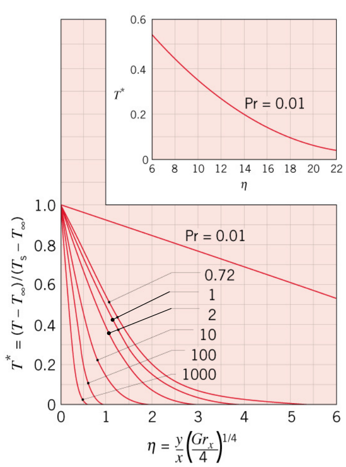
$$u = f'(\eta)\frac{2\nu}{x}\sqrt{Gr_x} \qquad (9\text{-}15)$$

similarity parameter

$$\eta = \frac{y}{x}\left(\frac{Gr_x}{4}\right)^{1/4} \qquad (9\text{-}13)$$



(a)



(b)

Laminar free convection on a vertical surface $\left(Ra_L < 10^9\right)$:

$$q_x'' = h_x\left(T_s - T_\infty\right)$$

$$Nu_x = \frac{h_x \cdot x}{k}$$

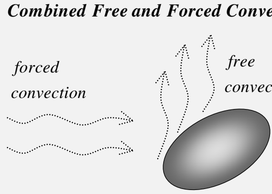
$$Nu_x = \left(\frac{Gr_x}{4}\right)^{1/4} \cdot g\left(Pr\right) \qquad (9.19)$$

$$g\left(Pr\right) = \frac{0.75\,Pr^{1/2}}{\left(0.609 + 1.221\,Pr^{1/2} + 1.238\,Pr\right)^{1/4}} \qquad (9.20)$$

$$q'' = \bar{h}\left(T_s - T_\infty\right)$$

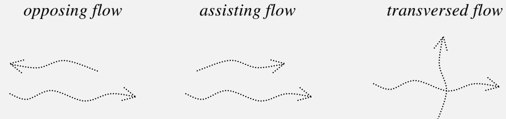
$$\overline{Nu}_L = \frac{\bar{h} \cdot L}{k} = \frac{4}{3}\left(\frac{Gr_L}{4}\right)^{1/4} \cdot g\left(Pr\right) = \frac{4}{3}Nu_L \qquad (9.21)$$

9.9 ***Combined Free and Forced Convection***



combined when

$$Gr_L \sim Re_L^2$$



opposing flow	assisting flow	transversed flow
$\boxed{-}$ $n = 3$	$\boxed{+}$ $n = 3$	$\boxed{+}$ $n = 7/2$ plate $n = 4$ cylinder, sphere

$$Nu^n = Nu^n_{forced} \boxed{\pm} Nu^n_{free}$$