Free Convection External

External Free Convection (immersed)

Properties at $T_f = \dfrac{T_s + T_\infty}{2}$

$$\overline{Nu_L} = \frac{\bar{h}L}{k} = C * Ra_L^n \quad (9.24)$$

For Laminar Flow C=.59 n=.25 for $10^4 \le Ra_L \le 10^9$

For Turbulent Flow C=.1 n=1/3 for $10^9 \le Ra_L \le 10^{13}$

$$Ra_L = Gr_L \Pr = \frac{g\beta(T_s - T_\infty)L^3}{\upsilon\alpha} \quad (9.25)$$

The Vertical Plate for T_s = Constant

$$\overline{Nu_L} = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{[1 + (0.492/\Pr)^{9/16}]^{8/27}}\right\}^2 \quad \text{For all Ra (9.26)}$$

$$\overline{Nu_L} = 0.68 + \frac{0.670 Ra_L^{1/4}}{[1 + (0.492/\Pr)^{9/16}]^{4/9}} \quad \text{For } Ra_L \le 10^9 \quad (9.27)$$

For vertical cylinders if $\dfrac{D}{L} \ge \dfrac{35}{Gr_L^{1/4}}$

For inclined plates $g \longrightarrow g\cos\theta$ for $\theta \le 60^\circ$

Horizontal Plates with $L_C \equiv \dfrac{A_s}{P}$

$$\overline{Nu_L} = 0.54 Ra_L^{1/4} \quad \text{For } 10^4 \le Ra_L \le 10^7 \quad (9.30)$$

$$\overline{Nu_L} = 0.15 Ra_L^{1/3} \quad \text{For } 10^7 \le Ra_L \le 10^{11} \quad (9.31)$$

Upper surface of hot plate or lower surface of cold plate.

$$\overline{Nu_L} = 0.27 Ra_L^{1/4} \quad \text{For } 10^5 \le Ra_L \le 10^{10} \quad (9.32)$$

Lower surface of hot plate or upper surface of cold plate.

Free Convection Continued

Horizontal Cylinder

$$\overline{Nu_D} = \frac{\bar{h}D}{k} = C * Ra_D^n \quad (9.33)$$

Ra	C	n
$10^{-10} \rightarrow 10^{-2}$	0.675	0.058
$10^{-2} \rightarrow 10^{2}$	1.02	0.148
$10^{2} \rightarrow 10^{4}$	0.85	0.188
$10^{4} \rightarrow 10^{7}$	0.48	0.25
$10^{7} \rightarrow 10^{12}$	0.125	0.333

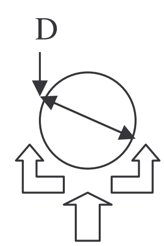
$$\overline{Nu_D} = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{[1+(0.559/\Pr)^{9/16}]^{8/27}}\right\}^2 \quad (9.34) \text{ For } Ra_D \le 10^{12}$$

Spheres

$$\overline{Nu_D} = 2 + \frac{0.589 Ra_D^{1/4}}{[1+(0.469/\Pr)^{9/16}]^{4/9}} \quad (9.35)$$

$\Pr \ge 0.7$

$Ra_D \le 10^{11}$



Free Convection continued

Plate Channels

Properties for isothermal at $T = \dfrac{T_s + T_\infty}{2}$ isoflux at $T = \dfrac{T_{s,L} + T_\infty}{2}$

$\overline{Nu_S} = \left(\dfrac{q/A}{T_s - T_\infty}\right)\left(\dfrac{S}{k}\right)$ (9.37) Isothermal definition $Ra_s = \dfrac{g\beta(T_s - T_\infty)S^3}{\nu\alpha}$ (9.38)

$\overline{Nu_S} = \dfrac{1}{24} Ra_s \left(\dfrac{S}{L}\right)\left\{1 - \exp\left[\dfrac{-35}{Ra_s(S/L)}\right]\right\}^{3/4}$ (9.36) symmetrically heated isothermal plates

EQ. 9.36 was developed for air as working fluid and rage is $10^{-1} \le \dfrac{S}{L} Ra_L \le 10^5$

$\overline{Nu_{S(fd)}} = \dfrac{1}{24} Ra_s \left(\dfrac{S}{L}\right)$ For fully developed flow (i.e. $\dfrac{S}{L} \to 0$) (9.39)

$\overline{Nu_{S(fd)}} = \dfrac{1}{12} Ra_s \left(\dfrac{S}{L}\right)$ For fully developed flow with insulated surface (9.40) ($\dfrac{S}{L} \to 0$)

Isoflux definition - $\overline{Nu_{S,L}} = \left(\dfrac{q''}{T_{s,L} - T_\infty}\right)\left(\dfrac{S}{k}\right)$ (9.41) $Ra_S^* = \dfrac{g\beta q_s'' S^4}{k\nu\alpha}$ (9.42)

$\overline{Nu_{S,L(fd)}} = 0.144[Ra_S^*(S/L)]^{1/2}$ (9.43)

$\overline{Nu_{S,L(fd)}} = 0.204[Ra_S^*(S/L)]^{1/2}$ (9.44) For asymmetric isoflux with one surface insulated.

$\overline{Nu_S} = \left[\dfrac{C_1}{(Ra_s S/L)^2} + \dfrac{C_2}{(Ra_s S/L)^{1/2}}\right]^{-1/2}$ (9.45) $Ra_s(orRa_s^*)\dfrac{S}{L} \le 10$

$\overline{Nu_S} = \left[\dfrac{C_1}{(Ra_S^* S/L)} + \dfrac{C_2}{(Ra_S^* S/L)^{2/5}}\right]^{-1/2}$ (9.46) $Ra_s(orRa_s^*)\dfrac{S}{L} \ge 100$

Free Convection Enclosures

Rectangular Cavity   Properties evaluated at $T = \dfrac{T_1 + T_2}{2}$

$$Ra_L = \frac{g\beta(T_1 - T_2)L^3}{\nu\alpha}$$ critical value for

$Ra_L \langle 1708$	Conduction h=k/L
$Ra_L \rangle 1708$	Convection

$$\overline{Nu_L} = \frac{\bar{h}L}{k} = .069 Ra_L^{1/3}\,\mathrm{Pr}^{.074} \quad \text{for } 3*10^5 \le Ra_L \le 7*10^9 \quad (9.49)$$

Vertical Rectangular cavity   *For Ra < 1000 conduction Nu=1. For Ra >1000 use correlations*

$$\overline{Nu_L} = 0.22\left(\frac{\mathrm{Pr}*Ra_L}{0.2+\mathrm{Pr}}\right)^{.028}\left(\frac{H}{L}\right)^{-1/4} \quad \text{For} \quad 2 \le \frac{H}{L} \le 10,\ \ \mathrm{Pr} \le 10^5,\ \ 10^3 \le Ra_L \le 10^{10} \quad (9.50)$$

$$\overline{Nu_L} = 0.18\left(\frac{\mathrm{Pr}*Ra_L}{0.2+\mathrm{Pr}}\right)^{0.29} \quad \text{For} \quad 1 \le \frac{H}{L} \le 2,\ \ 10^{-3} \le \mathrm{Pr} \le 10^5,\ \ 10^3 \le \frac{Ra_L*\mathrm{Pr}}{0.2+\mathrm{Pr}} \quad (9.51)$$

For Larger Aspect Ratios

$$\overline{Nu_L} = 0.42 Ra_L^{1/4}\,\mathrm{Pr}^{.012}\left(\frac{H}{L}\right)^{-0.3} \quad \text{For} \quad 10 \le \frac{H}{L} \le 40,\ \ 1 \le \mathrm{Pr} \le 2*10^4,\ \ 10^4 \le Ra_L \le 10^7 \quad (9.52)$$

$$\overline{Nu_L} = 0.046 Ra_L^{1/3} \quad \text{For} \quad 10 \le \frac{H}{L} \le 40,\ \ 1 \le \mathrm{Pr} \le 20,\ \ 10^6 \le Ra_L \le 10^9 \quad (9.53)$$

For inclined Cavities τ>0 and higher H/L>12 sec Eq. 9.54-9.57 on pg 590

Table 9.4 Critical Angle for inclined rectangular surface

H/L	1	3	6	12	>12
τ	25□	53□	60□	67□	70□

Concentric Cylinders *Properties eval. At* $T_m = \frac{T_i + T_o}{2}$

$$q = \frac{2L\pi k_{eff}}{\ln(r_o / r_i)}(T_i - T_o)$$ eq. (9.58)

$$\frac{k_{eff}}{k} = 0.386\left(\frac{\Pr}{0.861 + \Pr}\right)^{1/4} Ra_c^{1/4}$$ Eq. (9.59)

$0.7 \le \Pr \le 6000$
$Ra_c \le 10^7$

$$Ra_c^* = \frac{[\ln(D_o / D_i)]^4}{L^3 (D_i^{-3/5} + D_o^{-3/5})^5} Ra_L$$

$$L_c = \frac{2[\ln(r_o / r_i)]^{4/3}}{(r_i^{-3/5} + r_o^{-3/5})^{5/3}}$$ Eq. (9.60) (Length scale for Ra_c)

If $\frac{k_{eff}}{k} \le 1$ Then $k_{eff} = k$

Concentric spheres *Properties eval. At* $T_m = \frac{T_i + T_o}{2}$

$$q = \frac{4\pi k_{eff}(T_i - T_o)}{(1/r_i) - (1/r_o)}$$ Eq. (9.61)

$0.7 \le \Pr \le 4000$

$Ra_s \le 10^4$

$$\frac{k_{eff}}{k} = 0.74\left(\frac{\Pr}{0.861 + \Pr}\right)^{1/4} Ra_s^{1/4}$$ (9.62)

$$L_c = \frac{\left(\frac{1}{r_i} - \frac{1}{r_o}\right)^{4/3}}{2^{1/3}(r_i^{-7/5} + r_o^{-7/5})^{5/3}}$$ (9.63)

If $\frac{k_{eff}}{k} \le 1$ Then $k_{eff} = k$