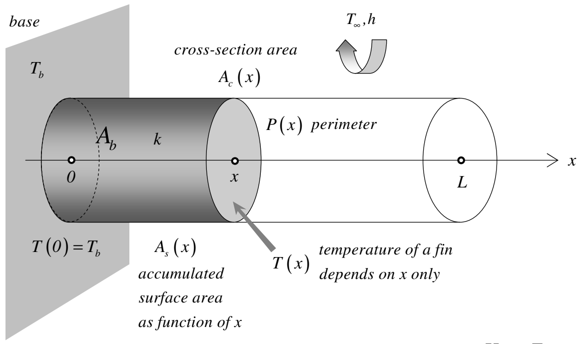
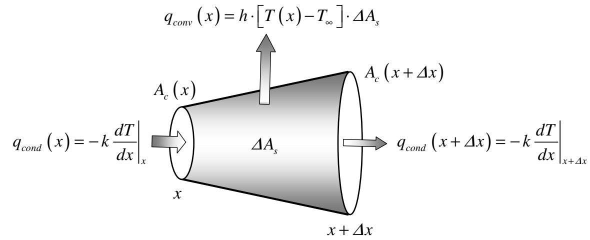
3.6 EXTENDED SURFACES – FINS

fins are used to enhance heat transfer



Control Volume

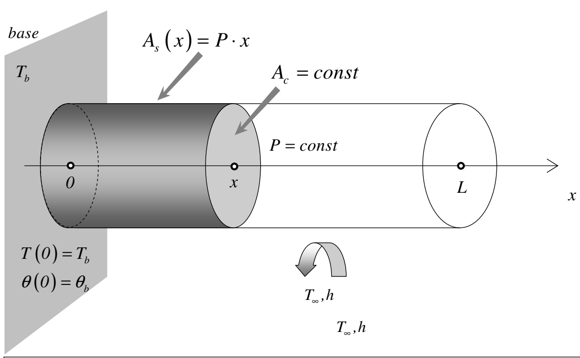


Energy balance: $q_{cond}(x) = q_{cond}(x+\Delta x) + q_{conv}$

Heat Equation for temperature distribution along a fin as a function of x

$$k \frac{d}{dx}\left(A_c \frac{dT}{dx}\right) = h(T - T_\infty)\frac{dA_s}{dx}$$

Fin of Uniform Cross-Section $A_c = const$



$\theta(x) = T(x) - T_\infty$ excess temperature

$\theta_b = T(0) - T_\infty$

Notation: $m = \sqrt{\frac{hP}{kA_c}}$

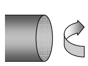
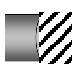
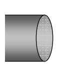
Heat Equation:

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

General solution:

$$\theta(x) = c_1 e^{-mx} + c_2 e^{mx}$$

TABLE 3.4 Temperature distribution and heat loss for fins of uniform cross section

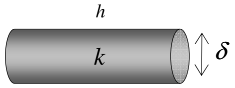
$x = L$	Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q_f
	A	Convection heat transfer: $h\theta(L) = -k\,d\theta/dx\|_{x=L}$	$\frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$ (3.70)	$M\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$ (3.72)
	B	Adiabatic $d\theta/dx\|_{x=L} = 0$	$\frac{\cosh m(L-x)}{\cosh mL}$ (3.75)	$M \tanh mL$ (3.76)
 T_L	C	Prescribed temperature: $\theta(L) = \theta_L$	$\frac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$ (3.77)	$M\frac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$ (3.78)
	D	Infinite fin ($L \to \infty$): $\theta(L) = 0$; $mL \geq 4.6$ for T, $mL \geq 2.65$ for q	e^{-mx} (3.79)	M (3.80)

$\theta \equiv T - T_\infty$ $\quad m^2 \equiv hP/kA_c$

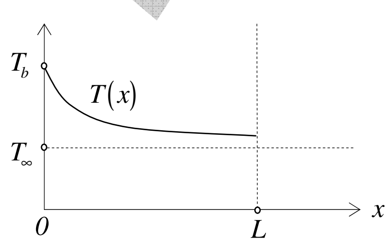
$\theta_b = \theta(0) = T_b - T_\infty$ $\quad M \equiv \sqrt{hPkA_c}\,\theta_b$

$$T(x) = T_\infty + (T_b - T_\infty) \cdot \theta/\theta_b$$

When elongated surface can be modeled as a fin?



$\frac{h\delta}{k} < 0.2$ *for error < 1%*



Rate of heat transfer from a fin:

$$q_f = q''(0) \cdot A_b = -k \cdot \frac{dT}{dx}\Big|_{x=0} \cdot A_b$$