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THE CROSS CORRELATION: AN ESSENTIAL MATHEMATICAL COMPUTATION FOR PARTICLE IMAGING VELOCIMETRY

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ABSTRACT

PIV systems are often used in fluid mechanics research because of their ability to gather simultaneous planar measurements of important fluid properties. Essential to the determination of velocity vectors in the plane is the ability to accurately ascertain the displacement of thousands of particles during a specified time step. This calculation is done internally by computer software with the use of a cross correlation calculation. In an effort to better understand the nature of the cross correlation, a study was performed of both one- and two-dimensional cross correlations using randomly generated data. The matrices were given a specified offset and the cross correlation was performed to prove that it can successfully determine the displacement between two data sets. This study made the cross correlation a more accessible concept and improved the understanding of how PIV software works.

NOMENCLATURE

C	cross correlation
dt	time step
dx	offset in the x-direction
dy	offset in the y-direction
r	matrix of particle positions: first image
s	matrix of particle positions: second image

subscript:

avg	average

INTRODUCTION

Fluid mechanics is an area of great interest in current research. Many methods have been introduced to aid in the gathering of data. Instruments such as pitot probes and hot-wires are extremely useful in determining fluid properties at individual points in a flow field. However, these instruments collect data only at discrete points in a limited number of locations in the flow field. Planar measurements of fluid characteristics are often employed to better understand the behavior of complicated flow fields.

One example of a planar measurement device is particle imaging velocimetry, or PIV. A PIV system works in the following way (Figure 1). First, thousands of tiny particles are introduced upstream of the flow field of interest. For wind tunnels, these particles are oil droplets on the order of 1-2μm in diameter. These particles travel downstream and are then illuminated in the area of interest by a sheet of light produced by a laser. The laser fires twice in rapid succession with a specified time delay between the laser pulses. The position of the oil particles in the flow field as illuminated by the laser is then recorded by either one or two digital cameras. Since both the displacements of the particles and the time delay between the two images are known, velocity vectors can be calculated throughout the entire plane. Thus, simultaneous planar measurements of velocity can be accurately determined even in complex flow fields.[1]

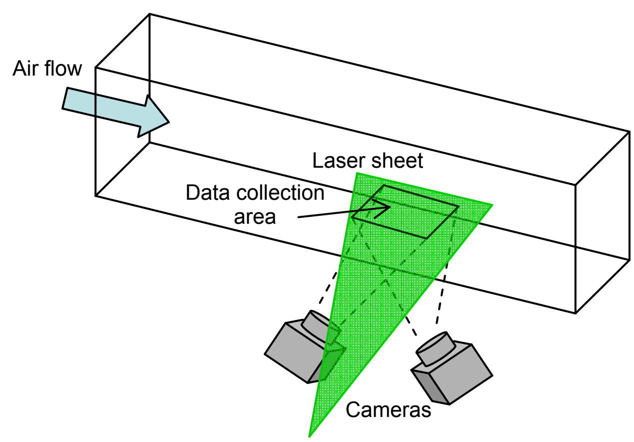


Figure 1: Schematic of PIV setup

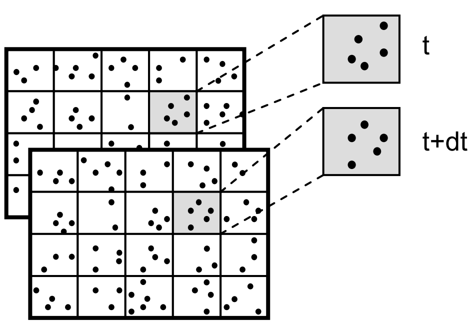
Due to the large volume of data that can be gathered by a PIV system, computer software is necessary in order to perform the velocity calculations. One of the essential functions of the software is to determine the movement (and likewise the displacement) of the oil particles between the first and second laser pulses. This procedure is performed with the use of a cross correlation function.

As PIV software has developed and improved, this cross correlation process has become imbedded in the software and is no longer user-specified. As a result, PIV software can become a "black box" where the user performs the experiment and receives the data output from the computer, but does not understand or appreciate the calculations done by the computer software. This lack of understanding could inhibit the usefulness of the experiment, as a thorough comprehension of the processes used to arrive at the final result is essential for proper interpretation of the data. It is the purpose of this paper to provide an explanation of the cross correlation function and how it works, so that the PIV system can be better understood and implemented in future fluid mechanics research.

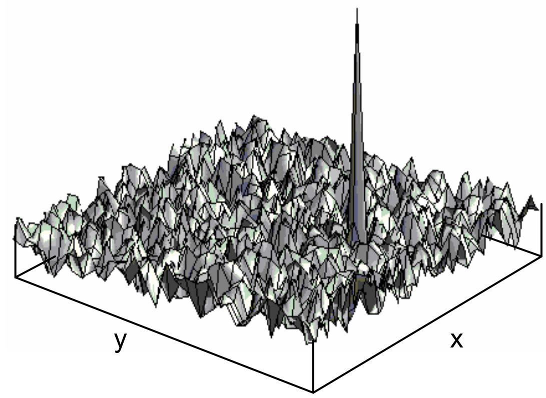
ANALYSIS

Description of the Cross Correlation

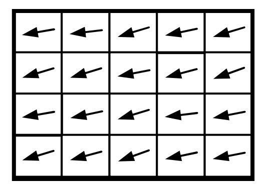
The PIV system computes a cross correlation by first extracting the particle images from the camera. These two images, separated by the time interval dt, contain the position of all the oil droplets. It must be determined which particle in the second image corresponds to the first image in order to determine the displacement of the particles and the direction of the flow. The image frames are subdivided into interrogation windows (there are 20 shown in Figure 2a) and the cross correlation is performed for each individual interrogation window. The results of the cross correlation are seen in Figure 2b. The large spike in the surface indicates the displacement of the particles in the x- and y-directions. In other words, if the first image is moved by the dx and dy specified by the cross correlation, the two images would be coincident. These displacements are used to calculate a velocity vector for each of the interrogation windows (Figure 2c).



a) Interrogation windows in the digital images taken at time t and t+dt



b) Result of cross correlation



c) Velocity vectors

Figure 2: Evaluation of PIV images using a cross correlation

One-Dimensional Example

Mathematically, the cross correlation, C, can be defined as

$$C_{1-D} = \frac{\sum_i [r(i) - r_{avg}][s(i-dx) - s_{avg}]}{\sqrt{\sum_i [r(i) - r_{avg}]^2}\sqrt{\sum_i [s(i-dx) - s_{avg}]^2}} \qquad (1)$$

where r and s represent two 1-D matrices, r_{avg} and s_{avg} represent the averaged values for the r and s matrices, and dx represents the offset distance between the two matrices.[3] Simply stated, the cross correlation computes the product of each value in the first matrix by a corresponding value in the second matrix, and then sums the result. If the values being multiplied are the same (or very close) the resulting product will be much higher than if the two values are unrelated. Therefore, the point where the cross correlation is at a maximum (as seen in Figure 2b) indicates the displacement necessary to bring both matrices into alignment.

The evaluation of the cross correlation equation is most simply illustrated using a one-dimensional example. Suppose there exists two series of data, r and y shown in Figure 3. r is randomly generated, and s is offset from r by a specific, but as yet unknown, time shift.

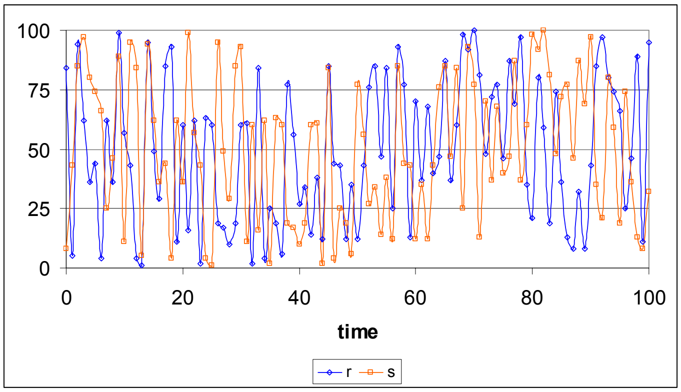


Figure 3: Data series r and s

The cross correlation between sets r and s is computed according to Eq. 1 starting at dx = 0 and successively increasing the offset distance. Figure 4 shows the results of the cross correlation for time offsets between 0 and 16.

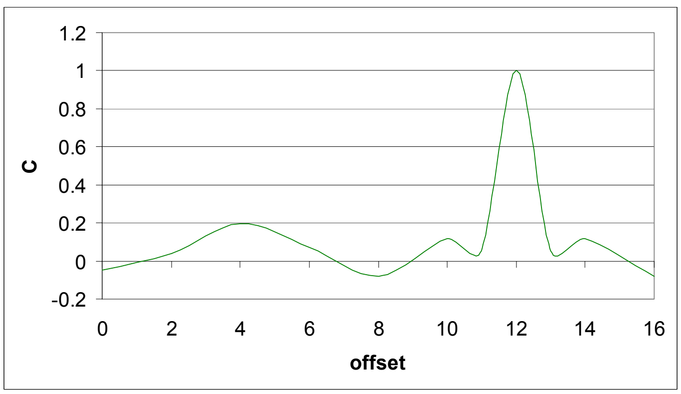


Figure 4: 1-D cross correlation between data series r and s

Figure 4 clearly shows a dramatic peak when s is offset by a time step of 12. This result can be graphically verified by plotting the original r set with s offset by 12, as shown in Figure 5. This basic example shows the effectiveness of the cross correlation in determining how to align two related sets of data.

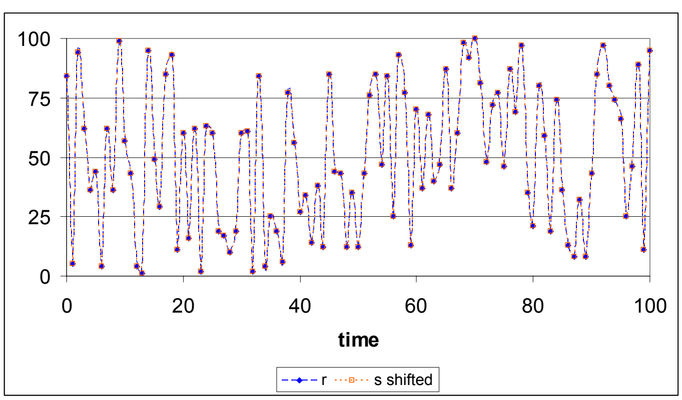


Figure 5: Plot of data series r and shifted s

Two-Dimensional Example

For the purposes of PIV measurements, however, the process becomes somewhat more complicated, as two-dimensional cross correlations are necessary to align data sets obtained in planes. The added dimension requires a modification to Eq. 1.

$$C_{2-D} = \frac{\sum_j \sum_i [r(i,j)-r_{avg}][s(i-dx,j)-s_{avg}]}{\sqrt{\sum_j \sum_i [r(i,j)-r_{avg}]^2}\sqrt{\sum_j \sum_i [s(i-dx,j)-s_{avg}]^2}} \quad (2)$$

Again, an example is provided to illustrate the usefulness of the two-dimensional cross correlation. r and s are now 2-D matrices, with r again generated randomly with values between 0 and 100. The s matrix is now offset in both the x- and y-directions. Contour plots of r and s are shown in Figure 6.

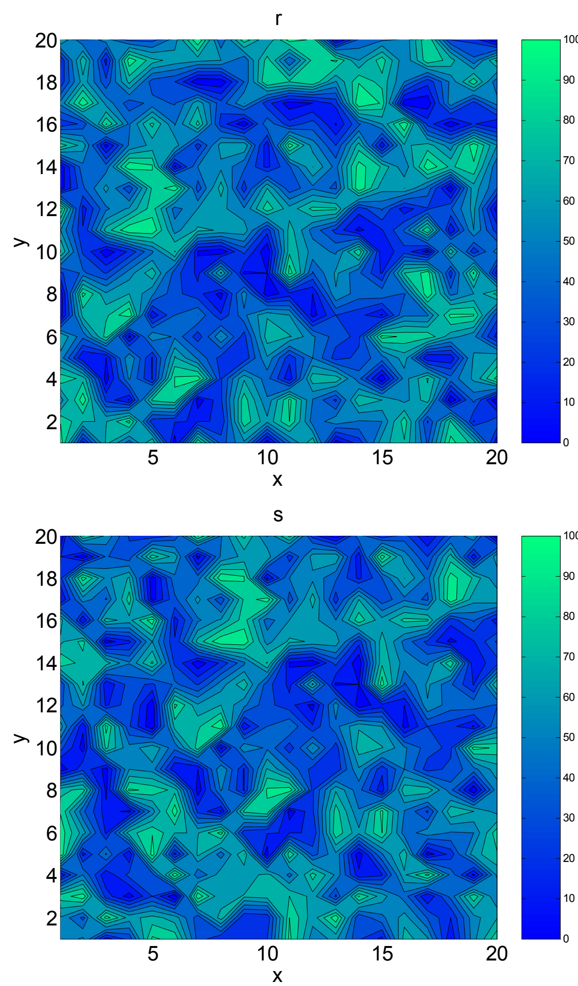


Figure 6: Contour plots two-dimensional data series r and s

The cross correlation is computed according to Eq. 2. The multiplications are performed systematically by shifting the s matrix successively by dx until it has been moved the entire x-length of the r matrix. Then s is returned to its original (unshifted) position and shifted over by dy, and the successive dx shifting is performed again. The process is repeated n this manner, and the products and sums are computed. The resulting cross correlation produced can be seen in Figure 7.

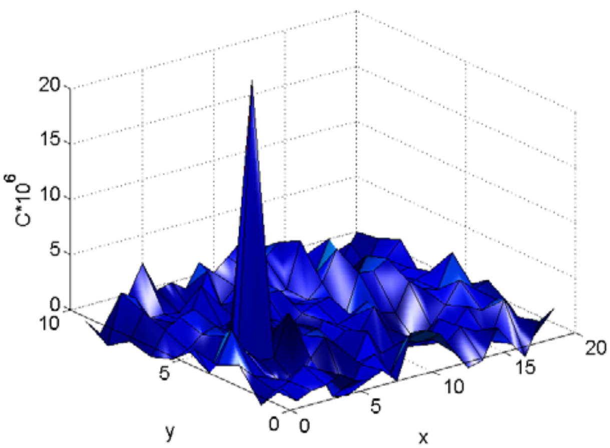


Figure 7: 2-D cross correlation between r and s

The spike seen in the surface plot of Figure 7 indicates that the data in s has been shifted by 5 units in both the x- and y-directions from the original r matrix. A close inspection of the r and s matrices in Figure 6 confirms this result.

CONCLUSIONS

The cross correlation is used internally in PIV software to determine the displacements of particles in a flow field. The information presented in this paper provided a definition of the cross correlation and explored how it functions in order to give a greater understanding of the workings of a PIV system. Two studies were performed for a one- and two-dimensional cross correlation. These studies were done using randomly generated data, and the results indicate that the cross correlation can be used successfully and reliably to determine the shift in two related matrices.

REFERENCES

[1] Bernard, P. S. and Wallace, J. M. Turbulent Flow: Analysis, Measurement, and Prediction. John Wiley & Sons Inc.: Hoboken, NJ, 2002, pp.84-93.

[2] User manual for DaVis FlowMaster Software by LaVision, Inc. in Göttingen, Germany, 2004, pp. 17-20.

[3] Website created by Paul Bourke at the Swinburne University of Technology in Austrailia. http://astronomy.swin.edu.au/~pbourke/analysis/correlate/.