Journal of Applied Engineering Mathematics

Volume 10, December 2023

ANALYSIS OF HEAT CONDUCTION ALONG TUBULAR SURFACES IN A CIRCULAR FINNED HEAT EXCHANGER: A FOURIER LAW INVESTIGATION

GEORGE ANKRAH, FRANCIS ANANE

Mechanical Engineering Department
Brigham Young University
Provo, Utah 84602
ga413@byu.edu, foanane@byu.edu

ABSTRACT

This study delves into the influence of thermal conductivity on the temperature profile along the tubes of a circular finned heat exchanger. Drawing insights from literature indicating that increased thermal conductivity diminishes temperature gradients, the research varies thermal conductivity using different materials. The investigation examines the ensuing impact on temperature distribution within the heat exchanger. Python was used to show the evolving temperature distribution at different time intervals. The results highlight how important material selection is for maximizing heat exchanger efficiency and how it might affect temperature distribution.

NOMENCLATURE

α	Thermal conductivity
$u(r,t)$	Temperature distribution in tube (K)
$u_s(r)$	Steady-state temperature distribution (K)
$U(r,t)$	Transient temperature distribution (K)
$\bar{h}$	Free convective heat transfer coefficient (W/m2·K)
t	Time (seconds)
u_0	Initial temperature of tube (K)
λ_n	Eigenvalue
r	Radial distance (m)
μ	Separation variable

INTRODUCTION

A circular finned-tube heat exchanger is a type of heat exchanger that utilizes a circular or annular arrangement of tubes with fins attached radially to enhance heat transfer. This design is often employed in applications where radial symmetry is beneficial, and it can be found in various industries, including HVAC systems, refrigeration, and electronic cooling. Hot fluid (such as steam or a liquid) flows through the tubes, and cold fluid (such as air or water) flows over the external surface of the tubes. Heat transfer occurs through conduction within the tube walls, convection from the fluid inside the tubes to the tube walls, and then again through convection from the tube walls to the fluid outside. The circular arrangement of tubes and fins provides radial symmetry, allowing for a more even distribution of heat transfer characteristics around the central axis.
By the Fourier law of heat conduction,

$$q = -\alpha\left(\frac{dT}{dx}\right)$$

Hence for attaining a constant amount of heat flux,

1) For higher thermal conductivity, the temperature gradient is smaller.
2) For lower thermal conductivity, the temperature gradient is higher.

METHOD

The problem was set up as an initial-boundary value problem with non-homogeneous boundary condition (thus two solutions, Steady State and Transient) and solved in polar coordinates.

$$\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} = a^2\frac{\partial u}{\partial t} \qquad u(r,t),\ r \in [0, r_1],\ t > 0$$

Initial Condtion $\quad U(r,0) = u_0(r)$

Boundary Condition $\quad k\frac{\partial u(r_1,t)}{\partial r} + hu(r_1,t) = f_1 \quad t > 0$

I. STEADY STATE SOLUTION

$$\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}=0$$

$$k\frac{\partial u_s(r_1,t)}{\partial r}+hu(r_1,t)=f_1$$

$$r^2\frac{\partial^2 u_s}{\partial r^2}+\mathrm{r}\frac{\partial u_s}{\partial r}=0$$

$$m^2+(1-1)m=0$$

$$u_s=c_1+c_2\ Inx$$

$$u_s=c_1+c_2\ Inx$$

$$k\frac{\partial u_s(r_1,t)}{\partial r}+hc_1=f_1 \qquad c_1=\frac{f_1}{h}$$

Steady State solution, $u_s=\frac{f_1}{h}$

II. TRANSIENT PROBLEM

1. Separation of variables

$$U(r,0)=\ u_0(r)-\ u_s$$

$$U(r,t)=\ R(r)T(t)$$

$U(r,t)\ \ bounded$ R (0) < ∞ bounded

$$k\frac{\partial U(r_1,t)}{\partial r}+hu(r_1,t)T(t)=0$$

$$\frac{R''}{R}+\frac{1}{r}\frac{R'}{R}=\frac{1}{\alpha}\frac{T'}{T}=\mu$$

2. Sturm-Liouville Problem

$$\frac{R''}{R}+\frac{1}{r}\frac{R'}{R}=\mu \qquad \mu=-\lambda^2$$

$$r^2R''+rR+[\lambda^2r^2-0^2]\ R=0$$

$$k\frac{\partial R(r_1)}{\partial r}+hR(r_1)=0$$

Eigenfunctions: $R_n(r)=J_0(\lambda_n r)$

Eigenvalues: λ_n are positive roots of the characteristic equation:

$$-\lambda J_1(\lambda r_1)+HJ_0(\lambda r_1)=0, \qquad H=h/k$$

Norm^2: $||R_n(r)||^2=\int_0^{r_1}J_0^2\,(\lambda_n r)rdr=\frac{r_1^2}{2}(\frac{H^2}{\lambda_n^2}+1)J_0^2(\lambda_n r_1)$

Weight Function: $p(r)=r$

3. Equation for T

$$T'-\alpha\mu T=0$$

$$T'+\alpha\lambda_n^2T=0$$

$$T_n(t)=e^{-\alpha\lambda_n^2t}$$

4. Transient Solution

$$u(r,t)=\sum_{n=1}^{\infty}a_nR_nT_n=\sum_{n=1}^{\infty}a_nJ_0(\lambda_n r)e^{-\alpha\lambda_n^2t}$$

$$U(r,0)=\ u_0(r)-u_s=\textstyle\sum_{n=1}^{\infty}a_nR_n$$

$$a_n=\frac{\int_0^{r_1}[u_0(r)-u_s]R_n\,rdr}{\int_0^{r_1}R_n^2\,(r)rdr}$$

III. SOLUTION

$$u(r,t)=u_s(r)+U(r,t)$$

RESULTS

The plot describes how temperature varies within the tubes over time, considering radial heat conduction. The equation provides insights into how heat is distributed and transferred within the cylindrical structure. It was observed that changing the material of the tube thus, using one with a higher thermal conductivity resulted in a more even distribution of the temperature profile over time.

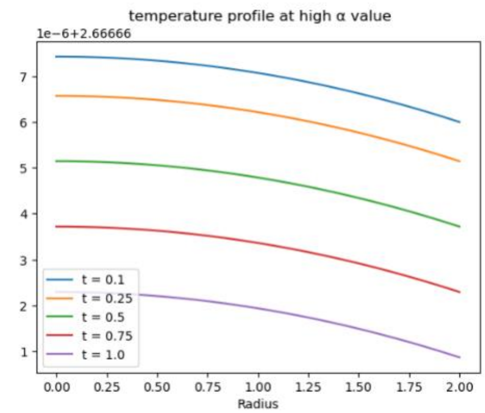

Figure 1: Temperature profile at large α

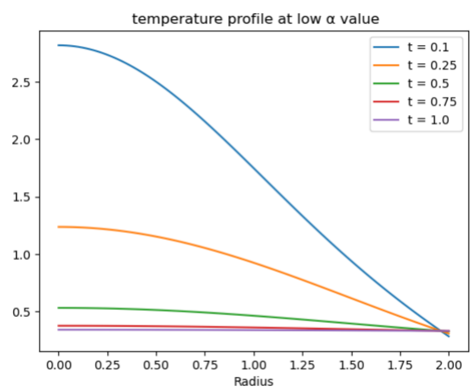

Figure 2: Temperature profile at small α

CONCLUSION

Despite not considering the heat flow in the angular position in our equation, the results clearly confirm the theory in literature that increasing thermal conductivity diminishes the temperature gradient along the tubes in the heat exchanger.

ACKNOWLEDGMENTS

We would like to show our appreciation to Dr. Vladimir Soloviev for his guidance and support during this project and also to our teaching assistant Noah Housley. Their insights and encouragement have been invaluable.

REFERENCES

[1] Heat and Mass Transfer. Yunus A. Cengel. McGraw-Hill Education, 2011. ISBN: 978118137253.

[2] Soloviev V. Integrated Engineering Mathematics: A Comprehensive Review and Advanced Material for Graduate Students and Professionals. 2023.

