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Feasibility of Firewalk Modeling

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Abstract

The epidermis of a firewalker's foot is modeled using the heat equation in a two dimensional domain. The required convection heat transfer coefficient to maintain a safe temperature at the depth of the epidermis is explored. Visualization techniques proved to be more difficult, but a correct model is developed and a 1-D simplified model is used to determine that a blood flow convective coefficient of 6000 W/m^2K would allow for safety when firewalking.

Nomenclature

ρ	Epidermis Density
c_p	Epidermis Specific Heat
h_a	Air Convection Coefficient
h_b	Blood Flow Convection Coefficient
k	Thermal Conductivity of skin
v	Point Source Velocity

Introduction

Firewalking is an extreme act of self discipline. In a firewalk, participants walk across coals reaching temperatures of 1000°F on their bare feet. This seemingly impossible feat has been a right of passage in several cultures, and remains practiced today. Participants hope to gain a level of control over their mind and body to overcome fears and obstacles. In this way, the fire is a metaphor for whatever they are facing in their lives. If they can walk across the coals, they can overcome anything.

Modeling the heat transfer into the body from the hot coals is a difficult task. The objective of this paper is to develop a robust model for the heat transfer from the coals into the epidermis of the sole of the foot in order to predict what the required internal blood flow convective coefficient is to maintain a safe temperature at the living skin cells deeper than the epidermis. However, due to the parameters involved in this situation, the limitations to such an approach will be explored. A model will be developed and validated on a simple case, and another one dimensional model will also be developed to help answer the question of what blood flow convective coefficient is required for safe firewalking.

1 Parameters of Interest

When considering the heat transfer from the coals to the sole of the foot, several parameters may come to mind. Perhaps some of the most compelling parameters when considering the seemingly impossible nature of skin cells surviving contact with such extreme temperatures are the thermal conductivity of the epidermis and the convection coefficient associated with blood flow. Some other parameters include the speed of the moving heat source (meant to model the moving point of contact when the foot rolls across the ground with each step), and the lengths associated with the domain. For this work, the following parameters, as summarized in Table 1 will be used as a starting point.

Table 1: Summary of several parameters used in firewalk model.

Parameter	Value
h_a	2 W/m^2K
h_b	300 W/m^2K
k	0.5 J/mK
v	1 m/s
L	270 mm
M	0.6 mm

2 Problem Statement

The heat transfer will be modeled using the heat equation given in Eq. 1.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1}$$

where $T(x, y, t)$ is the temperature at each point in the domain. The initial temperature of the domain is assumed to be body temperature T_b. In order to simplify the model, a new model is used such that the initial temperature is zero and the top boundary condition is homogeneous: $u(x, y, t) = T(x, y, t) - T_b$. The domain is shown in Figure 1. In the domain, h_a is the convection coefficient of the air below the foot, h_b is the convection coefficient due to blood flow, T_a is the temperature of the air under the foot, k is the conductivity of the skin, S_0 is the coal temperature of 1000°F or 811 K, and v is the velocity of the heat source representing the rolling of the foot along the hot coals.

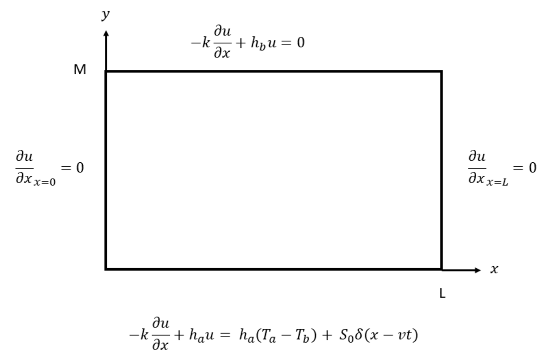

Figure 1: Domain of the model

The epidermis thickness is modeled to be 0.6mm per Lintzeri [2]. The length is modeled to be an average foot length of 270mm. The boundary conditions were chosen to model the extreme environment of the firewalk. As the epidermis thickness is so thin, the vertical edges of the domain are modeled to be insulated. The top and bottom edges are convective boundaries with an added source along the bottom edge. The source is modeled to be moving to the right, simulating a step taken on the hot coals.

3 Solution

The Finite Fourier Transform as well as the Laplace Transform are used in the solution. All solutions to the nine possible boundary condition combinations are known for a 2-D domain. It is recognized that the model has Neumann-Neumann and Robin-Robin boundary conditions. The eigenvalues, eigenfunctions, norms, and operational properties can be found in [3].

Using a Fourier transform in x and y, and a Laplace transform, the derivatives were eliminated and the equation could be solved algebraically for temperature. After performing an inverse transform, the transformed model u could be defined as follows in Equation 2.

$$\overline{\overline{u}} = \frac{\lambda_m h_a \alpha}{k}(T_a - T_b)e^{-\alpha(\mu_n^2+\lambda_m^2)t} + S_0\alpha \left[\frac{\alpha(\mu_n^2+\lambda_m^2)cos(\mu_n vt) + \mu_n v sin(\mu_n vt) - \alpha(\mu_n^2+\lambda_m^2)e^{\alpha(\mu_n^2+\lambda_m^2)t}}{\alpha^2(\mu_n^2+\lambda_m^2)^2 + \mu_n^2 v^2} \right] \tag{2}$$

With Eq. 2 we can write the solution modeling the temperature at any point in our domain and at any time as follows in Eq. 3.

$$T(x,y,t) = T_b + \sum_{m=1}\sum_{n=0} \frac{X_n Y_m}{||X_n||^2||Y_m||^2}\overline{\overline{u}} \tag{3}$$

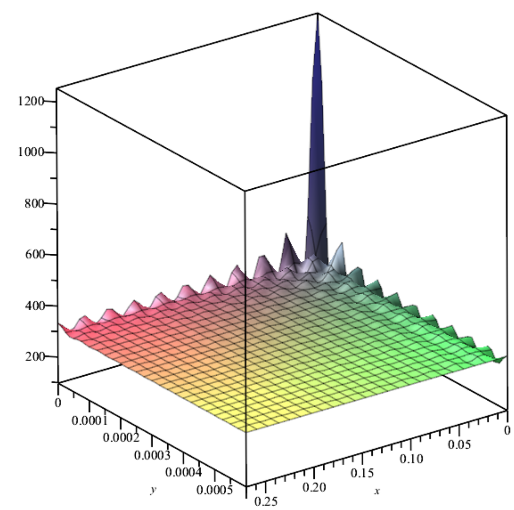

Figure 2: Model of Epidermis temperature at t=0

4 Results

Maple was used as the primary visualization software. It was at this step in the process when I started having difficulty understanding what was going on in my model. In the solution above, 30 terms were used in the summation. Due to the fact that a Dirac Delta function was being used in my moving source term, and due to the fact that I am using a finite number of summation terms, oscillations appear to be present in my visualized solution. This can be seen in Figure 2.

The oscillations seen are not physical, but a result of the modeling approach. Such issues led to difficulty in interpreting the result of the animation depicting a moving point source. I was convinced that my solution was incorrect until I varied the parameters to model a simple metal flat plate exposed to similar boundary conditions. A contour plot of this result is shown in Figure 3.

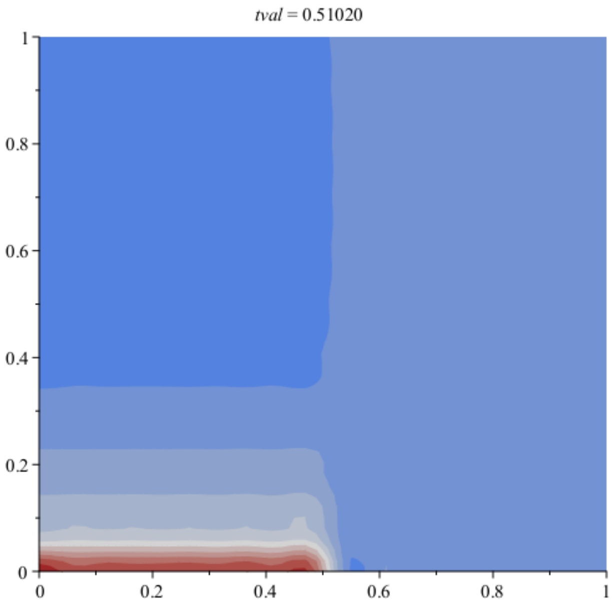

Figure 3: A metal flat plate domain at t = 0.51 showing a temperature distribution

This figure does not represent the domain of a firewalker's foot, but rather is a closer representation of an aluminum plate exposed to similar boundary conditions as the firewalker. This figure is only meant to illustrate for the reader what the expected heat transfer contours would look like if the parameters in this firewalk model were not as extreme. As can be seen, the heat source has moved from the left side of the domain to about halfway. Heat can be seen transferring upward though the domain and being removed from the convective boundary condition on top. This is the same phenomenon that occurs within the outermost layer of skin on our bodies. It was hoped that this model would inform us what heat transfer coefficient was required to maintain a safe temperature at the top of the domain in a firewalker's foot. Unfortunately, the results are inconclusive. Further effort is required to establish a better form of visualizing the solution in order to answer this question. Another possible path forward would be to explore other methods of applying the moving source other than the Dirac Delta function.

In an effort to gain some insight into the required convective coefficient for pulling away enough heat to spare the skin cells at the top of the domain, the problem statement was rethought. Instead of modeling a foot rolling across the hot coals, a static foot standing on the hot coals was investigated. This change allowed us to model the domain as a one dimensional thickness of the epidermis as the heat would be applied to the entire bottom surface of the foot. It also eliminated the need for a convective boundary condition with a moving point source on the bottom surface. Instead, a constant temerature (Dirichlet) boundary condition can be used, with the temperature equal to the coal temperature.

It must be understood that this change is a new problem, not a just a rework of the original one. The purpose of this change is to gain some understanding for what our h_b could be to allow for safety when standing still on hot coals. The simplification to the problem allows us to solve it directly using separation of variables as it is now a one dimensional differential equation with non-homogeneous boundary conditions. The solution is summarized as follows.

The steady state solution is shown in Eq. 4

$$T_s(y) = \frac{h_b(T_b - S)}{k + h_b M} y + S \tag{4}$$

where the parameters are the same as those listed above except for h_b which will be discussed further. After finding the transient solution, the final one dimensional model can be given as shown in Eq. 5.

$$T(y,t) = T_s(y) + \sum_{m=1} \frac{\int_0^M (T_b - T_s(y)) Y_m(y) dy}{\int_0^M Y_m^2(y) dy} Y_m(y) e^{-\alpha \lambda_m^2 t} \tag{5}$$

This equation is plotted for several points in time along the thickness from 0 to M in Figure 4.

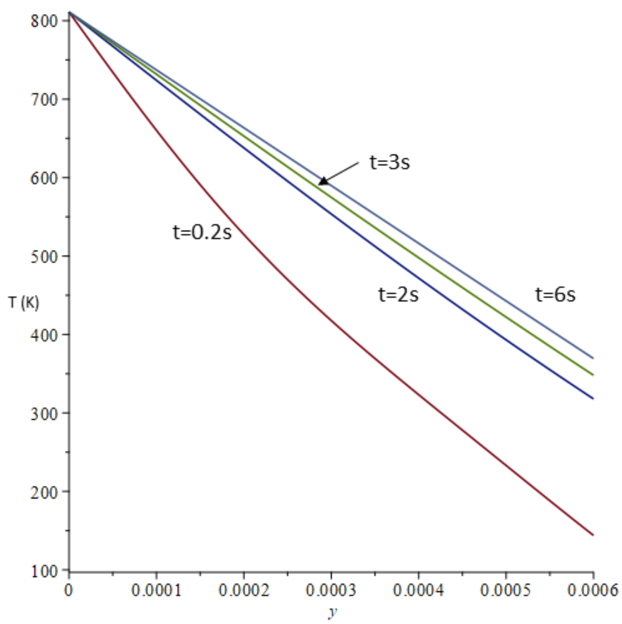

Figure 4: 1-D model of epidermis at t = 0.2 - 6 seconds.

As can be seen in the figure, the heat transfer quickly approaches its steady state after only about 5.5 s. This model was explored in order to determine what convective coefficient h_b is required to maintain a safe temperature at the upper thickness of the epidermis ($y = M = 0.6mm$). A ”safe temperature” is defined to be below 100°C to avoid scalding and death of skin cells deeper than the epidermis. It was found that a convective coefficient of 6000 W/m^2K is sufficient to maintaint the steady state temperature at $y = M$ below 372 K (99°C). This is the convective coefficient used in the plot in Figure 4.

A convective coefficient of 6000 W/m^2K is well within the range of possible convective coefficients for forced convection using liquids (such as in blood flow) per Bergman and Lavine in [1]. Thus, the one dimensional model does inform us that it may be possible to safely walk across hot coals without burning and damaging living skin cells deeper than the epidermis.

Conclusions

A correct model for the heat transfer in the epidermis of a firewalker with a moving point source was developed, however, the visualization technique employed was unable to satisfactorily answer the larger question of the required convective coefficient to safely firewalk. Using the Dirac Delta function with our given parameters resulted in a visualization that could not lead to a helpful answer. However, the solution form was demonstrated to be correct when applied to a simpler case of a flat plate under similar boundary conditions to the firewalker’s foot. Thus, the solution is correct, but the visualization of that solution for the firewalker requires more time and careful thought.

A one dimensional model was able to satisfactorily prove that for a static firewalker, a convective coefficient of 6000 W/m^2K is sufficient to pull heat away from the epidermis and prevent damage to skin cells above, essentially avoiding a burn. This provides useful insight into the feasibility of firewalking safely. Participants can know that preventing injury is well within the realm of possibility so long as their heart is able to pump enough blood to produce this convective coefficient.

It is recommended that further study be put into the modeling of this phenomenon. Other source term applications can be explored such as a parabolic or sinusoidal functions. Perhaps using a simpler analytical function to model the moving source would result in a more helpful visualization.

Overall, this paper was able to demonstrate that it is feasible to model the heat transfer occurring within the foot of a firewalker, but more careful visualization techniques are required to develop a transient animation of the heat moving in the epidermis when steps are taken.

Acknowledgements

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References

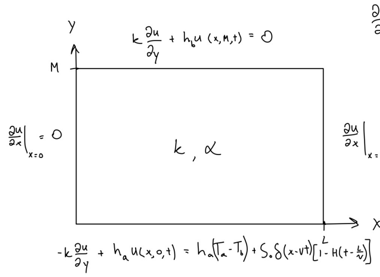
[1] T.L. Bergman and A.S. Lavine. *Fundamentals of Heat and Mass Transfer*. 8th ed. 2019.

[2] D.A. Lintzeri et al. "Epidermal thickness in healthy humans: a systematic review and meta-analysis". In: *Journal of the European Academy of Dermatology and Venereology* (2022).

[3] Vladimir Solovjev. *Integrated Engineering Mathematics*. 2021.

Appendix

A full written solution is included below:



$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$T(x,y,0) = T_b$$

$$u(x,y,t) = T(x,y,t) - T_b$$

$$\frac{\partial T}{\partial y} = \frac{\partial u}{\partial y}$$

$$-k\frac{\partial T}{\partial y} + h_a T(x,0,t) = h_a T_a + f(x,t)$$

$$\hookrightarrow -k\frac{\partial u}{\partial y} + h_a u = h_a(T_a - T_b) + f(x,t)$$

X: N-N

Eigenvalues: $\mu_0 = 0$ $\quad \mu_n = \frac{n\pi}{L}$ $n=1,2,\dots$

Eigenfunctions: $X_0 = 1$ $\quad X_n = \cos(\mu_n x)$

$\|X_0\|^2 = L$ $n=0$ $\quad \|X_n\|^2 = \frac{L}{2}$ $n=1,2\dots$

Operational Property: $-\mu_n^2 \bar{T}_n - 0(X_n(0)) + 0(X_n(L)) = -\mu_n^2 \bar{T}_n$

Y: R-R $\quad$ let $H_1 = \frac{h_a}{k}$, $H_2 = \frac{h_b}{k}$

Eigenvalues are positive roots of:

$$(H_1H_2 - \lambda^2)\sin(\lambda M) + (H_1 + H_2)\lambda\cos(\lambda M) = 0$$

Eigenfunction: $Y_m = \lambda_m\cos(\lambda_m y) + H_1\sin(\lambda_m y)$ $\quad$ Note $Y_m(0) = \lambda_m$

$$\|Y_m\|^2 = \frac{\lambda_m^2 + H_1^2}{2}\left[M + \frac{H_2}{\lambda_m^2 + H_2^2}\right] + \frac{H_1}{2}$$

Operational Property: $-\lambda_m^2\bar{T} + \frac{h_a(T_a - T_b) + S_o\delta(x-vt)\left[1 - H\left(t - \frac{L}{v}\right)\right]}{k}(\lambda_m)$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{\alpha}\frac{\partial u}{\partial t}$$

Fourier transform with respect to y

$$\frac{\partial^2 \bar{u}}{\partial x^2} - \lambda_m^2\bar{u} + \frac{h_a(T_a - T_b) + S_o\delta(x-vt)\left[1 - H\left(t - \frac{L}{v}\right)\right]}{k}\lambda_m = \frac{1}{\alpha}\frac{\partial \bar{u}}{\partial t}$$

Fourier transform with respect to x

$$-\mu_n^2\bar{\bar{u}} - \lambda_m^2\bar{\bar{u}} + \frac{\lambda_m}{k}\left[h_a(T_a - T_b) + S_o\left[1 - H\left(t - \frac{L}{v}\right)\right]\cos(\mu_n v t)\right] = \frac{1}{\alpha}\frac{\partial \bar{\bar{u}}}{\partial t}$$

Laplace transform $\quad$ Neglect heaviside for now...

$$-\mu_n^2\hat{\bar{\bar{u}}} - \lambda_m^2\hat{\bar{\bar{u}}} + \frac{\lambda_m}{k}\left[h_a(T_a - T_b) + S_o\frac{s}{s^2 + (\mu_n V)^2}\right] = \frac{1}{\alpha}s\hat{\bar{\bar{u}}}$$

$$\hat{\bar{\bar{u}}}\left[\mu_n^2 + \lambda_m^2 + \frac{s}{\alpha}\right] = \frac{\lambda_m}{k}\left[h_a(T_a - T_b) + S_o\frac{s}{s^2 + (\mu_n V)^2}\right]$$

$$\hat{\bar{\bar{u}}} = \frac{1}{\frac{s}{\alpha} + (\mu_n^2 + \lambda_m^2)}\frac{\lambda_m}{k}h_a(T_a - T_b) + \frac{1}{\frac{s}{\alpha} + (\mu_n^2 + \lambda_m^2)}S_o\frac{s}{s^2 + (\mu_n V)^2}$$

Note: Multiply RHS by $\frac{\alpha}{\alpha}$

$$\hat{\bar{\bar{u}}} = \frac{\lambda_m h_a \alpha}{k}(T_a - T_b)\frac{1}{s - (-\alpha(\mu_n^2 + \lambda_m^2))} + S_o\alpha\frac{s}{s^2 + (\mu_n V)^2}\frac{1}{s - (-\alpha(\mu_n^2 + \lambda_m^2))}$$

Inverse Laplace

$$\bar{\bar{u}} = \frac{\lambda_m h_a \alpha}{k}(T_a - T_b)e^{-\alpha(\mu_n^2 + \lambda_m^2)t} + S_o\alpha\left[\frac{\alpha(\mu_n^2 + \lambda_m^2)\cos(\mu_n V t) + \mu_n V\sin(\mu_n V t) - \alpha(\mu_n^2 + \lambda_m^2)e^{-\alpha(\mu_n^2 + \lambda_m^2)t}}{\alpha^2(\mu_n^2 + \lambda_m^2)^2 + \mu_n^2 V^2}\right]$$

$$T(x,y,t) = T_b + \sum_{m=1}\sum_{n=0}\frac{X_n Y_m}{\|X_n\|^2\|Y_m\|^2}\bar{\bar{u}}$$