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Modeling Rogue Waves in the Middle of a Hurricane

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Abstract

Hurricanes happen each year across the world. Most people know that the eye of the Hurricane one will find the most relaxed conditions with the least amount of wind. This is true on land; however, on sea the eye of the hurricane can be extremely dangerous. Despite the relatively calm winds, waves maintain their energy and some of the largest waves can be found in the eye (insert reference here). Waves can create constructive interference leading to the phenomenon known as ”rogue waves.” This paper explores the plausible creation and motion of these waves using a mathematical model.

Nomenclature

Significant Wave Height: Wave height based on energy of the wave
$u(x, t)$: Height of the ocean wave
v: Wave propagation speed

Introduction

Hurricanes are an extremely dangerous natural disaster that occur every year. Rogue waves are a very interesting phenomenon that are known to occur in the eye of these storms. But what are rogue waves? They are waves that are at least twice as high as the significant wave height of the waves around them [1]. These waves are still not fully understood.

The purpose of this paper is to use a simple mathematical model to explore what might create these rogue waves. Many theories think of the rogue wave as a superposition of other waves in the area. The issue is to determine what conditions might create this superposition that leads to a rogue wave.

Methods

A normal hurricane will create waves in 3-dimensional space; however, the principle behind rogue waves can be idealized to just 1-dimensional space. As a result, the underlying equation of motion used for this paper is the 1-dimensional wave equation.

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} + S(x, t) \tag{1}$$

To explore the superposition of waves, the test environment will be in the deep ocean in the eye of the hurricane. This will mean that the domain is infinite.

$$-\infty < x < \infty \tag{2}$$

At the eye of a hurricane, there are ideally no winds (not necessarily true, but a decent approximation), therefore the source term $S(x, t) = 0$. Since the boundaries are infinite, the wave will only have initial conditions as follows:

$$u(x,0) = f(x) \tag{3}$$

$$\frac{\partial u}{\partial x}(x,0) = 0 \tag{4}$$

The solution to this PDE is simply d'Alembert's solution where the derivative term is 0:

$$u(x,t) = \frac{1}{2}\left(f(x - vt)\right) \tag{5}$$

Since this is a linear PDE, any linear combination of solutions will work. For the purpose of this paper, two waves were modeled and determined for what conditions a rogue wave will occur. Two cases were determined. The first case is when both waves are going opposite directions, at some point in time they will superimpose and create a rogue wave. For this case a base wave used a similar method to other studies [2] by representing the wave as a Gaussian distribution.

$$u_1(x,t) = \frac{1}{2}\left(Ae^{-(\frac{x}{A}-vt)^2} - Ae^{-(\frac{x}{A}-vt-2)^2}\right) \tag{6}$$

where A represents the wave amplitude and the 2nd term represents a negative Gaussian distribution shifted to the right representing the trough of the wave. The 2nd wave was placed some distance L away from the first wave going the opposite direction with $-v$ velocity.

$$u_2(x,t) = \frac{1}{2}\left(-Ae^{-(\frac{x}{A}+vt-L)^2} + Ae^{-(\frac{x}{A}+vt-2-L)^2}\right) \tag{7}$$

The second case was determined if two waves were going in the save direction but with different speeds. Hence equation 6 is kept but equation 7 is modified to have a speed of λv where λ represents some scalar velocity. For both cases $u(x,t)$ is represented by the superposition:

$$u(x,t) = u_1(x,t) + u_2(x,t) \tag{8}$$

Results

Case 1

For this case, the wave height was determined to be around 30 feet based off of data recorded by NOAA

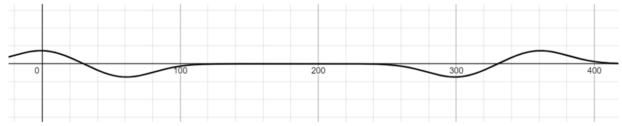

Figure 1: Case 1 when time is 0

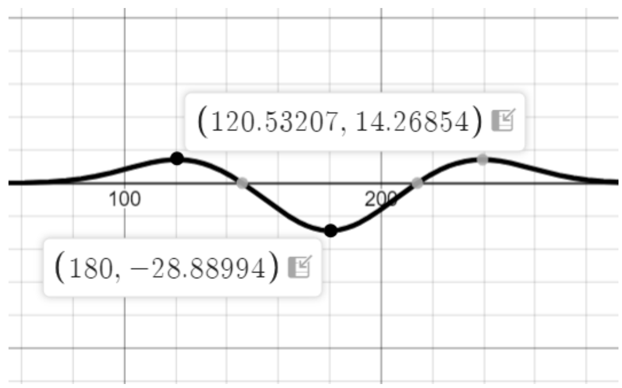

Figure 2: Case 1 when time is 4.1

of the highest average significant wave height within a hurricane [3]. This led to an initial wave distribution as follows where L was set to 10: 1 The waves ended up superimposing at time 4.1 which led to a maximum wave height of about 45 feet from height to trough. 2, followed by This led to an increase in wave height of about 12 feet which is about 1.5x the original wave height.

Case 2

This case involved setting the wave's close together but at slightly different speeds. The units of v don't necessarily matter, rather it is the relative speed that will determine the amount of time the waves stay superimposed. L was set to 7 to space the waves slightly closer (units are very arbitrary and not needed). The relative speed λ was set at 0.3. 3

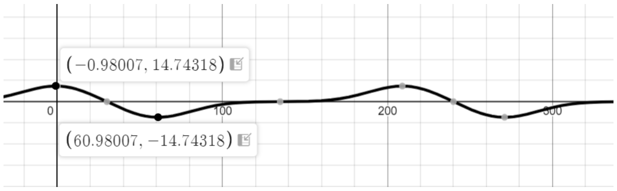

Figure 3: Case 2 when time is 0

This led to the the waves superimposing to a height of 60 feet which was exactly 2x the original significant wave height. This occured at time = 10 and lasted for roughly 2 time units. 4

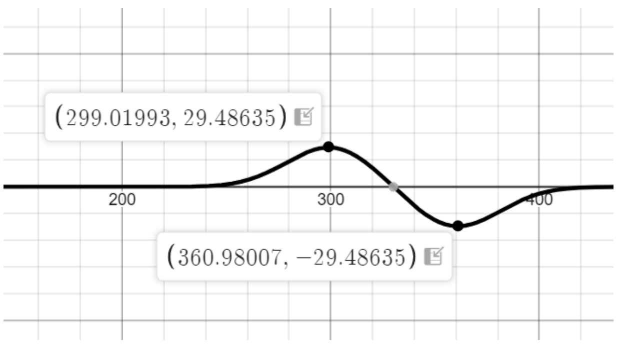

Figure 4: Case 2 when time is 10

Conclusions

The results show that the most likely case for a rogue wave being created is from waves moving in the same direction at different speeds. The waves moving in opposite directions doesn't create a high enough wave height to be counted as a rogue wave (2x significant wave height). The waves moving in the same direction at slightly different speeds, does however create a rogue wave.

There are many simplifications used in this model and all non-linearities are ignored which could pose significant effects when it comes to rogue waves. The author acknowledges that this phenomenon is very complicated and can create things not captured in this study. However, this study does add to the conversation by bringing up the question, is there a phenomenon that allows waves to have differing propagation speeds in the deep ocean? If so, does this factor into rogue waves? These questions require further research.

Acknowledgements

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[3] *What happens to wave height during a hurricane?*

