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TRANSIENT HEAT CONDUCTION IN STEEL UNDER EXTREME SURFACE HEAT FLUX: A LIGHTSABER EXAMPLE

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ABSTRACT

Transient heat transfer problems can be very important when considering manufacturing and engineering operations. This paper models the interaction of a high-energy plasma beam – commonly known as a "lightsaber," with a semi-infinite steel wall. The system is analyzed using the transient heat transfer equation and uses Laplace transforms coupled with the Neumann boundary conditions to obtain the analytical solution. The results show that the plasma beam can increase the temperature of the steel to its melting point, illustrating its effectiveness in manufacturing processes such as welding and cutting.

NOMENCLATURE

T	Temperature (K)
T_i	Initial Temperature (K)
t	Time (s)
x	Depth into material (mm)
α	Thermal diffusivity (m^2/s)
k	Thermal conductivity (W/m·K)
q_0	Surface heat flux (W/m^2)
ρ	Density (kg/m^3)
c	Specific heat capacity (J/(kg·K)

INTRODUCTION

Modern manufacturing and engineering processes often use high-energy thermal interactions with solid materials, including laser cutting, plasma machining and welding. Each of these processes involve large heat fluxes for very short amounts of time. This leads to large temperature gradients. The heat transfer in these situations can be well represented by the transient heat transfer equation.

In this paper, a "lightsaber" is used as a source of concentrated heat. This heat source acts like a high energy plasma. This thermal response of a semi-infinite steel wall when subjected to the high-intensity heating by the "lightsaber." By modeling the steel wall as a semi-infinite solid, the transient heat transfer equation can be used to approximate the situation under ideal conditions. The objective of this paper is to derive the temperature profile of the steel wall when exposed to the "lightsaber" beam.

METHODS

Initial and Boundary conditions:

The temperature profile in the semi-infinite steel wall can be modeled by the transient heat transfer equation:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \quad x > 0, t > 0 \tag{1}$$

Where α is defined as:

$$\alpha = \frac{k}{(\rho c)} \tag{2}$$

For a semi-infinite solid starting at ambient temperature, the initial conditions (IC) are as follows:

$$IC: T(x,0) = T_i\,, T(\infty,t) = T_i \tag{3}$$

The boundary condition for this situation is well represented by the Neumann boundary condition:

$$-k\frac{\partial T}{\partial x}(0,t) = q_0 \quad for\ (t > 0) \tag{4}$$

Solution Method:

To solve the transient heat transfer equation, Laplace transforms was selected because they can easily handle the Neumann boundary condition and results in familiar error functions in the solution. Let:

$$u(x,t) = T(x,t) - T_i \tag{5}$$

Substituting u(x,t) in for T, equations 1 and 4 can be rewritten as:

$$\frac{\partial u}{\partial t} = \alpha\frac{\partial^2 u}{\partial x^2} \quad x > 0, t > 0 \tag{6}$$

And

$$-k\frac{\partial u}{\partial x}(0,t) = q_0 \quad for\ (t > 0) \tag{7}$$

The Laplace transform is defined as:

$$\mathcal{L}\{u(x,t)\}(s) = U(x,s\_ = \int_0^\infty e^{-st}u(x,t)\,dt \tag{8}$$

From this definition and after rearrangement, the transformed PDE can be rewritten as:

$$\frac{d^2U}{dx^2} - \frac{s}{\alpha}U = 0 \tag{9}$$

The solution to this ODE, when $m = \frac{\sqrt{s}}{\alpha}$ is:

$$U(x,s) = A(s)e^{-mx} \tag{10}$$

The boundary condition from equation 7 is applied, where the Laplace transform of the flux is defined as $Q(s) = \frac{q_0}{s}$, resulting in the following:

$$A(s) = \frac{q_0\sqrt{\alpha}}{ks^{\frac{3}{2}}} \tag{11}$$

Thus:

$$U(x,s) = \frac{q_0\sqrt{\alpha}e^{-\alpha\sqrt{s}}}{ks^{\frac{3}{2}}} \tag{12}$$

After applying the reverse Laplace transform we obtain:

$$u(x,t) = \frac{q_0}{k}\left[2\sqrt{\frac{\alpha t}{\pi}}e^{-\frac{x^2}{4\alpha t}} - x\,erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)\right] \tag{13}$$

We then use $T = T_i + u$ to obtain the following closed-form solution:

$$T(x,t) = T_i + \frac{q_0}{k}\left[2\sqrt{\frac{\alpha t}{\pi}}e^{-\frac{x^2}{4\alpha t}} - x\,erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)\right] \tag{14}$$

RESULTS

The solution in Equation 14 was evaluated numerically to simulate a steel wall exposed to the “lightsaber” beam. Using material properties of steel, and approximating the heat flux q_0 as 22 MW/m², we obtain the following plot.

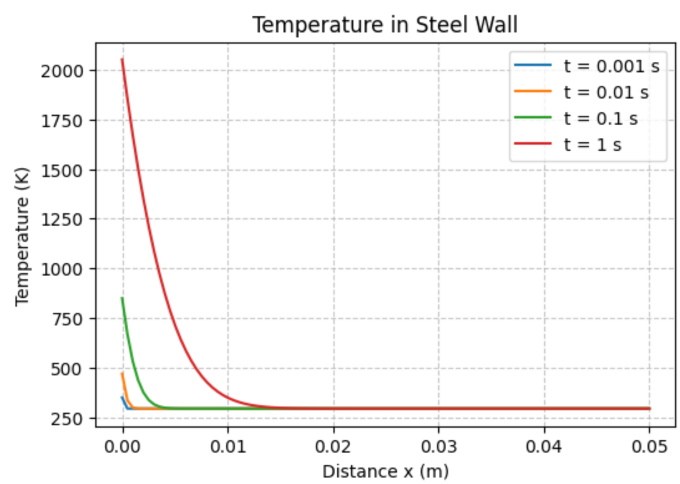

Figure 1. plot of temperature (K) vs distance into a steel wall (m) for t = 0.001, 0.01, 0.1, and 1,where T_i = 20°C, q_0 = 22 MW/m², k = 54 W/(m·K), ρ = 7850 kg/m³ and c = 470 J/(kg·K)

As shown in Figure 1, the temperature of the surface of the metal can reach temperatures high enough to melt the metal wall (>2000 K) who’s melting point is approximately 1700-1800 K, in less than 1 second making it ideal for industrial manufacturing and engineering processes.

CONCLUSIONS

The temperature gradient of a semi-infinite steel wall when exposed to a high temperature plasma beam, such as a “lightsaber” was found by using Laplace transforms to solve the transient heat transfer equation. This illustration can help to conceptualize how time can impact the magnitude of heat transfer effects within a solid surface. It also shows how high temperature beams can be used as cutting and welding tools in manufacturing by illustrating the impact that high energy, low duration pulses of heat can be used to reach high temperatures quickly and consistently, thus making them suitable and practical for industrial processes.

APPENDIX

Derivation of Analytical Solution

Defining Equation

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \qquad x > 0,\ t > 0$$

Initial conditions

$$T(x,0) = T_i \qquad T(\infty, t) = T_i$$

Boundary Conditions surface flux (Neumann)

$$-k \left.\frac{\partial T}{\partial x}\right|_{x=0} = q_0 \qquad \text{for } t > 0$$

let $v(x,t) = T(x,t) - T_i$ $\qquad v(x,0) = 0$, $v(\infty, t) = 0$

$$\frac{\partial v}{\partial t} = \alpha \frac{\partial^2 v}{\partial x^2} \qquad \text{where } -k \left.\frac{\partial v}{\partial x}\right|_{x=0} = q_0$$

$$\alpha = \frac{k}{\rho c}$$

Laplace Transform

$$\mathcal{L}\{v(x,t)\}(s) = V(x,s) = \int_0^\infty e^{-st} v(x,t)\,dt$$

$$sV(x,s) = \alpha \frac{d^2 V}{dx^2}(x,s)$$

$$\frac{d^2 V}{dx^2} - \frac{s}{\alpha} V = 0 \qquad \text{let } m = \sqrt{\frac{s}{\alpha}}$$

only take positive section

$$V(x,s) = A(s) e^{-mx}$$

$$\mathcal{L}\{q_0\} = Q(s) = \frac{q_0}{s}$$

$$\therefore -k \left.\frac{\partial V}{\partial x}\right|_{x=0} = Q(s) = \frac{q_0}{s}$$

$$\frac{\partial V}{\partial x} = -A(s) m e^{-mx}$$

$$-k(-Am) = kAm = \frac{q_0}{s} \longrightarrow A(s) = \frac{q_0}{ksm} = \frac{q_0\sqrt{\alpha}}{k s^{3/2}}$$

$$\therefore V(x,s) = A(s) e^{-mx} = \frac{q_0\sqrt{\alpha}}{k s^{3/2}} e^{-x\sqrt{\frac{s}{\alpha}}}$$

$$a = \frac{x}{\sqrt{\alpha}}$$

$$V(x,s) = \frac{q_0\sqrt{\alpha}}{k s^{3/2}} e^{-a\sqrt{s}}$$

$$\mathcal{L}^{-1}\{V(x,s)\}(t) = 2\sqrt{\frac{t}{\pi}}\, e^{-\frac{a^2}{4t}} - a\,\mathrm{erfc}\left(\frac{a}{2\sqrt{t}}\right)$$

were $\mathrm{erfc}(z) = 1 - \mathrm{erf}(z) = \frac{2}{\sqrt{\pi}} \int_z^\infty e^{-s^2}\,ds$

$$\therefore v(x,t) = \frac{q_0\sqrt{\alpha}}{k}\left[2\sqrt{\frac{t}{\pi}}\, e^{-\frac{x^2}{4\alpha t}} - \frac{x}{\sqrt{\alpha}}\,\mathrm{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)\right]$$

$$= \frac{q_0}{k}\left[2\sqrt{\frac{t\alpha}{\pi}}\, \exp\left(-\frac{x^2}{4\alpha t}\right) - x\,\mathrm{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)\right]$$

$v = T - T_i$

$$T(x,t) = T_i + \frac{q_0}{k}\left[2\sqrt{\frac{t\alpha}{\pi}}\, \exp\left(-\frac{x^2}{4\alpha t}\right) - x\,\mathrm{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)\right]$$

Python script used to solve numerically

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.special import erfc

# --- 1. Define Variables (Change these as needed) ---
Ti = 20.0+273.15      # K
q0 = 22*10**6     # Surface heat flux (W/m^2)
k = 54        # Thermal conductivity (W/(m K))
rho = 7850  #kg/m^3
c = 470     #J/(kg K)
alpha = k/(rho*c)

# Define the spatial domain (x)
# Generating 100 points from x=0 to x=0.1 meters (10 cm)
x = np.linspace(0, 0.05, 100)

# Define 5 different time snapshots (in seconds)
times = [0.001,0.01,0.1,1]

# --- 2. Calculation and Plotting ---
plt.figure(figsize=(6, 4))

for t in times:
    # Avoid division by zero if t=0
    if t <= 0:
        continue

    # Calculate the grouping inside the brackets
    # Term 1: 2 * sqrt(alpha*t/pi) * exp(...)
    term1 = 2 * np.sqrt((alpha * t) / np.pi) * np.exp(-(x**2) / (4 * alpha * t))

    # Term 2: x * erfc(...)
    term2 = x * erfc(x / (2 * np.sqrt(alpha * t)))

    # Combine terms according to the equation
    T = Ti + (q0 / k) * (term1 - term2)

    # Plot the curve for this time t
    plt.plot(x, T, label=f't = {t} s')

# --- 3. Formatting the Graph ---

plt.title(f'Temperature in Steel Wall')
plt.xlabel('Distance x (m)')
plt.ylabel('Temperature (K)')
plt.legend()
plt.grid(True, which='both', linestyle='--', alpha=0.7)

# Show the plot
plt.show()
```

