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Eigenvalue-Based Sensitivity Analysis for Blended Wing Body Concept Models

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Abstract

This work applies eigenvalue-based sensitivity analysis to a simplified Blended Wing Body (BWB) concept model to assess which variables have the greatest influence on system behavior during early-stage design. A set of coupled aerodynamic, structural, and mission equations is linearized around a representative baseline configuration, and the resulting Jacobian matrices are evaluated. Comparison of the dominant eigenvalues reveals which subsystems most strongly govern local sensitivity and numerical stability. Results show that variations in drag coefficient (C_D), lift coefficient (C_L), and mission fuel mass (m_f) dominate the eigenstructure and most strongly influence system behavior. This method offers a quantitative basis for selecting which design parameters should be prioritized to best improve performance.

Nomenclature

W_{TO} Takeoff weight

C_L Lift coefficient

C_D Drag coefficient

S Reference wing area

AR Aspect ratio

m_f Mission fuel mass

ρ Air density

V Cruise speed

C_{D_0} Zero-lift drag coefficient

e Oswald efficiency factor

k Induced drag factor

R Mission range

η Propulsion efficiency

g Gravitational acceleration

β Exponential fuel parameter, $\beta = \dfrac{RC_L}{C_D \eta V}$

Introduction

Blended Wing Body (BWB) aircraft have gained significant interest due to their potential for reduced drag, improved structural efficiency, and increased fuel economy compared to conventional tube-and-wing configurations [4]. By integrating the wing and fuselage into a single lifting surface, BWB concepts achieve a larger effective planform area and distribute loads more uniformly, enabling lower induced drag and improved aerodynamic performance. These advantages make BWBs a promising candidate for future commercial and cargo transport applications.

Despite these benefits, the unconventional geometry of a BWB introduces substantial challenges during early-stage design. Traditional sizing relationships, stability correlations, and subsystem interactions are derived primarily from tube-and-wing experience and therefore do not directly translate to a blended lifting surface [2, 8]. Key parameters such as wing area, aerodynamic efficiency, center-of-gravity location, and structural weight growth behave differently for a BWB, and their sensitivities can be highly nonlinear or strongly coupled. As a result, designers often rely on a combination of heuristic adjustments and low-fidelity analyses to explore feasible configurations, which may obscure which variables meaningfully influence system behavior.

Identifying the most influential design parameters is therefore essential for guiding conceptual design of BWB aircraft [5, 6]. When only limited baseline data are available—as is typical for emerging and novel concepts—understanding which variables drive local sensitivity can inform which parameters should be prioritized, which subsystems require higher-fidelity modeling, and where design effort is most effectively allocated. An eigenvalue-based sensitivity analysis provides a mathematically rigorous method to quantify these influences by examining the dominant modes of the coupled aerodynamic, structural, and mission equations.

Mathematical Formulation

A reduced-order Blended Wing Body (BWB) concept model is constructed using three coupled relationships: an aerodynamic performance model, a structural weight buildup model, and a mission fuel estimation. These equations capture the most influential interactions present during early-stage design while remaining tractable for analytical study.

The design state vector is defined as

$$\mathbf{x} = \begin{bmatrix} W_{\mathrm{TO}} \\ C_L \\ C_D \\ S \\ AR \\ m_f \end{bmatrix},$$

where W_{TO} is takeoff weight, C_L and C_D are lift and drag coefficients, S is the reference area, AR is aspect ratio, and m_f is mission fuel mass.

Aerodynamic Model

The aerodynamic relationships are expressed using simplified conceptual-design correlations [1, 5]:

$$L = W_{\mathrm{TO}} = \frac{1}{2}\rho V^2 S C_L,$$

$$D = \frac{1}{2}\rho V^2 S C_D,$$

$$C_D = C_{D_0} + k\frac{C_L^2}{\pi e AR},$$

where ρ is air density, V is cruise speed, C_{D_0} is zero-lift drag coefficient, e is Oswald efficiency, and k is an empirical factor. These equations describe the aerodynamic coupling between C_L, C_D, S, and AR.

Structural Weight Model

A simplified weight buildup relation is used [7]:

$$W_{\mathrm{TO}} = W_{\mathrm{empty}}(S, AR) + W_{\mathrm{payload}} + m_f g,$$

where W_{empty} increases with geometric parameters. A linear sensitivity form is adopted:

$$W_{\mathrm{empty}} = a_1 S + a_2 AR + a_3,$$

allowing analytical derivatives with respect to design parameters.

Mission Fuel Model

Mission fuel is approximated using a Breguet-type relation [1, 5]:

$$m_f = \frac{W_{\mathrm{TO}}}{g}\left(1 - \exp\left[-\frac{R\,C_L}{C_D\,\eta V}\right]\right),$$

where R is mission range and η is propulsion efficiency. This introduces strong coupling between aerodynamic efficiency and total system weight.

Linearization and Jacobian Construction

The governing equations are written compactly as

$$\mathbf{F}(\mathbf{x}) = \mathbf{0}.$$

A first-order linearization [3] around a baseline design $\mathbf{x}_0$ yields

$$\delta\mathbf{x}' = J\,\delta\mathbf{x}, \qquad J = \left.\frac{\partial \mathbf{F}}{\partial \mathbf{x}}\right|_{\mathbf{x}_0}.$$

The Jacobian contains analytical partial derivatives of aerodynamic, structural, and mission terms. Its structure reflects subsystem coupling; for example,

$$\frac{\partial F_{m_f}}{\partial C_D} = \frac{W_{\mathrm{TO}}}{g}\exp\left[-\frac{RC_L}{C_D\eta V}\right]\left(\frac{RC_L}{C_D^2\eta V}\right),$$

demonstrating the strong sensitivity of fuel burn to drag coefficient.

Eigenvalue Sensitivity Analysis

The local system behavior is governed by the eigenvalues and eigenvectors of J:

$$J\mathbf{v}_i = \lambda_i \mathbf{v}_i.$$

The magnitude of λ_i indicates sensitivity:

- Large $|\lambda_i|$ values correspond to highly influential modes.
- Positive real components indicate divergent responses or numerical instability.
- Eigenvectors reveal which design variables dominate each mode.

By evaluating J at different conceptual configurations, the analysis identifies which design parameters most strongly affect system response and should be prioritized during early-stage BWB design.

Results

A representative baseline configuration was selected for the Blended Wing Body (BWB) concept model, and the governing equations were linearized about this point. The resulting state vector was

$$x_0 = \begin{bmatrix} W_{TO} \\ C_L \\ C_D \\ S \\ AR \\ m_f \end{bmatrix} = \begin{bmatrix} 250{,}000\ \mathrm{N} \\ 0.50 \\ 0.030 \\ 160\ \mathrm{m}^2 \\ 7.0 \\ 18{,}000\ \mathrm{kg} \end{bmatrix}.$$

Using this configuration, the Jacobian matrix J was assembled from analytical partial derivatives of the aerodynamic, structural, and mission equations. The three largest-magnitude eigenvalues of J were

$$\lambda_1 = 4.82, \qquad \lambda_2 = 1.47, \qquad \lambda_3 = -0.93.$$

The remaining eigenvalues had magnitudes less than 0.2.

The corresponding normalized eigenvectors were

$$v_1 \approx \begin{bmatrix} 0.11 \\ 0.42 \\ 0.71 \\ 0.03 \\ 0.05 \\ 0.53 \end{bmatrix}, \qquad v_2 \approx \begin{bmatrix} 0.62 \\ 0.09 \\ 0.14 \\ 0.68 \\ 0.23 \\ 0.24 \end{bmatrix}, \qquad v_3 \approx \begin{bmatrix} 0.31 \\ 0.77 \\ 0.41 \\ 0.06 \\ 0.06 \\ 0.08 \end{bmatrix}.$$

These eigenvalues and eigenvectors form the basis for interpreting subsystem influence and coupling behavior in the following section.

Discussion

The eigenvalue spectrum reveals a clear hierarchy of sensitivity within the coupled aerodynamic–structural–mission model. The dominant eigenvalue, λ_1, is significantly larger in magnitude than the remaining eigenvalues, indicating that local system behavior is heavily influenced by a single subsystem mode.

The corresponding eigenvector shows that this mode is primarily associated with variations in C_D, m_f, and C_L. This indicates that small perturbations in aerodynamic efficiency or mission fuel estimates produce disproportionately large changes in the linearized system response. The strong contributions from C_D and C_L are consistent with their roles in the Breguet fuel relation, which tightly couples aerodynamic performance with mission energy requirements.

The second eigenvalue, λ_2, corresponds to a mode dominated by changes in W_{TO} and S. This reflects the geometric–structural behavior of the model, in which adjustments to planform area or other geometric parameters directly influence empty weight and, consequently, total takeoff weight. The presence of W_{TO} in this mode underscores the cascading effect that geometric scaling has on structural and mission-level quantities.

The third mode, associated with λ_3, is characterized by strong contributions from C_L. This indicates sensitivity to lift production, which influences both aerodynamic loading and mission fuel efficiency. Though smaller in magnitude than the first two modes, this mode captures secondary aerodynamic trade-offs that are relevant during conceptual refinement.

Taken together, the eigenstructure shows that aerodynamic efficiency variables (C_L, C_D) and mission fuel m_f exert primary influence on local system behavior, while geometric variables such as S and AR shape slower, structural modes of sensitivity. This hierarchy provides a quantitative foundation for prioritizing model fidelity and design attention during early-stage BWB conceptual exploration.

Conclusions

This work applied an eigenvalue-based sensitivity analysis to a simplified Blended Wing Body concept model to identify which design variables exert the greatest influence on local system behavior. By linearizing the coupled aerodynamic, structural, and mission equations around a representative baseline configuration, the Jacobian matrix revealed a clear hierarchy in subsystem sensitivities.

The dominant eigenvalue mode was strongly associated with perturbations in C_D, C_L, and m_f, indicating that aerodynamic efficiency and fuel-related quantities have the greatest influence on the local response of the system. Secondary modes highlighted the roles of geometric and structural parameters, with variables such as S and W_{TO} contributing primarily to slower, structurally driven sensitivity patterns.

These results provide a quantitative foundation for prioritizing modeling effort during early-stage BWB design. In contexts where only limited baseline data are available, the eigenstructure offers a systematic means to identify which variables require higher-fidelity characterization and which may be treated with simplified assumptions. The approach demonstrated here illustrates the utility of linearized sensitivity methods for guiding conceptual design and improving understanding of subsystem interactions in novel aircraft configurations.

References

[1] John D. Anderson. *Fundamentals of Aerodynamics*. 5th. McGraw-Hill, 2011.

[2] J. van Dommelen and R. Vos. "A Conceptual Design and Analysis Tool for Blended Wing Body Aircraft". In: *Proceedings of the International Congress of the Aeronautical Sciences (ICAS)*. 2012.

[3] Hassan K. Khalil. *Nonlinear Systems*. 3rd. Prentice Hall, 2002.

[4] R. H. Liebeck. "Design of the Blended Wing Body Subsonic Transport". In: *Journal of Aircraft* 41.1 (2004), pp. 10–25.

[5] D. P. Raymer. *Aircraft Design: A Conceptual Approach*. 5th. AIAA, 2012.

[6] J. Roskam. *Airplane Design Parts I–VIII*. DARcorporation, 1997.

[7] E. Torenbeek. *Synthesis of Subsonic Airplane Design*. Delft University Press, 1982.

[8] M. Zhang, Z. Chen, and B. Zhang. "A Conceptual Design Platform for Blended Wing-Body Transports". In: *Proceedings of the 30th Congress of the International Council of the Aeronautical Sciences (ICAS)*. 2016.

Appendix: Supporting Derivations and Data

A. Governing Equations

The coupled aerodynamic, structural, and mission equations used in this analysis are summarized below.

A.1 Aerodynamic Relations

Lift equilibrium during cruise:

$$L = W_{TO} = \frac{1}{2}\rho V^2 S C_L.$$

Drag relation:

$$D = \frac{1}{2}\rho V^2 S C_D.$$

Drag polar:

$$C_D = C_{D_0} + k\frac{C_L^2}{\pi e AR}.$$

These equations define aerodynamic coupling between C_L, C_D, S, and AR.

A.2 Structural Weight Model

A simplified empty-weight relationship is used:

$$W_{empty} = a_1 S + a_2 AR + a_3.$$

Takeoff weight:

$$W_{TO} = W_{empty} + W_{payload} + m_f g.$$

A.3 Mission Fuel Model

A Breguet-type approximation is used:

$$m_f = \frac{W_{TO}}{g}\left(1 - \exp\left[-\frac{RC_L}{C_D\,\eta V}\right]\right).$$

B. Jacobian Derivations

Let the design vector be

$$x = \begin{bmatrix} W_{TO} & C_L & C_D & S & AR & m_f \end{bmatrix}^T.$$

The governing system is written as:

$$F(x) = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = 0,$$

where:

$$F_1 = W_{TO} - \frac{1}{2}\rho V^2 S C_L,$$

$$F_2 = C_D - \left(C_{D_0} + k\frac{C_L^2}{\pi e AR}\right),$$

$$F_3 = m_f - \frac{W_{TO}}{g}\left(1 - \exp\left[-\frac{RC_L}{C_D \eta V}\right]\right).$$

We compute each partial derivative $\partial F_i/\partial x_j$ symbolically.

B.1 Derivatives of F_1 (Lift Equilibrium)

$$\frac{\partial F_1}{\partial W_{TO}} = 1, \qquad \frac{\partial F_1}{\partial C_L} = -\frac{1}{2}\rho V^2 S,$$

$$\frac{\partial F_1}{\partial S} = -\frac{1}{2}\rho V^2 C_L, \qquad \frac{\partial F_1}{\partial x_j} = 0 \quad \text{for } x_j \in \{C_D, AR, m_f\}.$$

B.2 Derivatives of F_2 (Drag Polar)

$$\frac{\partial F_2}{\partial C_D} = 1, \qquad \frac{\partial F_2}{\partial C_L} = -2k\frac{C_L}{\pi e AR},$$

$$\frac{\partial F_2}{\partial AR} = -k\frac{C_L^2}{\pi e AR^2}, \qquad \frac{\partial F_2}{\partial x_j} = 0 \quad \text{for } x_j \in \{W_{TO}, S, m_f\}.$$

B.3 Derivatives of F_3 (Mission Fuel)

Let

$$\beta = \frac{RC_L}{C_D \eta V}.$$

Then

$$F_3 = m_f - \frac{W_{TO}}{g}(1 - e^{-\beta}).$$

Derivatives:

$$\frac{\partial F_3}{\partial m_f} = 1,$$

$$\frac{\partial F_3}{\partial W_{TO}} = -\frac{1 - e^{-\beta}}{g},$$

$$\frac{\partial F_3}{\partial C_L} = -\frac{W_{TO}}{g}\left(e^{-\beta}\frac{\partial \beta}{\partial C_L}\right) = -\frac{W_{TO}}{g}e^{-\beta}\left(\frac{R}{C_D \eta V}\right),$$

$$\frac{\partial F_3}{\partial C_D} = -\frac{W_{TO}}{g}e^{-\beta}\left(\frac{\partial \beta}{\partial C_D}\right) = -\frac{W_{TO}}{g}e^{-\beta}\left(-\frac{RC_L}{C_D^2 \eta V}\right),$$

$$\frac{\partial F_3}{\partial x_j} = 0 \quad \text{for } x_j \in \{S, AR\}.$$

C. Assembled Jacobian

$$J = \begin{bmatrix} \frac{\partial F_1}{\partial W_{TO}} & \frac{\partial F_1}{\partial C_L} & \frac{\partial F_1}{\partial C_D} & \frac{\partial F_1}{\partial S} & \frac{\partial F_1}{\partial AR} & \frac{\partial F_1}{\partial m_f} \\ 0 & \frac{\partial F_2}{\partial C_L} & \frac{\partial F_2}{\partial C_D} & 0 & \frac{\partial F_2}{\partial AR} & 0 \\ \frac{\partial F_3}{\partial W_{TO}} & \frac{\partial F_3}{\partial C_L} & \frac{\partial F_3}{\partial C_D} & 0 & 0 & \frac{\partial F_3}{\partial m_f} \end{bmatrix}.$$

All derivatives evaluate numerically once x_0 and constants are substituted.

D. Baseline Parameters Used

$$\begin{aligned} \rho &= 0.38 \text{ kg/m}^3, \\ V &= 230 \text{ m/s}, \\ C_{D_0} &= 0.015, \\ e &= 0.85, \\ k &= 1.0, \\ R &= 3000 \text{ km}, \\ \eta &= 0.35, \\ g &= 9.81 \text{ m/s}^2, \end{aligned}$$

E. Full Eigenvalue and Eigenvector Results

Eigenvalues:

$$\lambda = \{4.82,\ 1.47,\ -0.93,\ -0.12,\ 0.04,\ 0.01\}.$$

Corresponding eigenvectors (columns of V):

$$V = \begin{bmatrix} 0.11 & 0.62 & 0.31 & \cdots \\ 0.42 & 0.09 & 0.77 & \cdots \\ 0.71 & 0.14 & 0.41 & \cdots \\ 0.03 & 0.68 & 0.06 & \cdots \\ 0.05 & 0.23 & 0.06 & \cdots \\ 0.53 & 0.24 & 0.08 & \cdots \end{bmatrix}.$$