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PRESSURE WAVES CHANGING MEDIA

Matthew Linnell

Mechanical Engineering Department
Brigham Young University
Provo, Utah 84602
Ml553@byu.edu

ABSTRACT

Pressure waves move at varying speeds depending on the properties of the medium they are moving through. This means that their behavior can change when they meet a boundary between two different mediums. This work sets up a mathematical framework for predicting the behavior of waves at these boundaries based on the material properties of both mediums and the input wave. The results show that there will be a portion of the input wave that will be reflected by the boundary and a portion that will transmit across the boundary. The wave is divided based on the material properties.

NOMENCLATURE

P = Pressure (Pa)
t = Time (s)
c = speed of sound (m/s)
λ = wavelength (m)
R = Reflection coefficient
T = Transmission coefficient

INTRODUCTION

Pressure waves are a constant part of our lives. They include a wide range of things, but the one that we are most familiar with is sound. Sound waves are generated by a large number of sources and can be simply modeled as pressure waves moving through some medium.

An interesting property of pressure waves is that they move at a different speed depending on the medium. This is an intrinsic property of pressure waves because they exist because of the interactions of the medium's particles. This means that, unlike electromagnetic waves moving through a vacuum, there is no "universal" speed of sound. This means that any medium that a pressure/sound wave is moving through must have a "speed of sound" defined.

There are many instances where it is both interesting and useful to understand how a pressure wave behaves as it hops from one medium to another. This includes cases as common as the human ear as it captures sound waves from the air. These pressure waves must transition from the air to the eardrum in order for the ear to "hear" the sound. This work focuses on understanding this behavior as well as any other pressure waves.

Problem Setup

Pressure waves are typically represented mathematically with the wave equation:

1) $$\frac{d^2P}{dt^2} = c^2 \left(\frac{d^2P}{dx^2} + \frac{d^2P}{dy^2} + \frac{d^2P}{dz^2}\right)$$

Equation 1 is the three dimensional wave equation. For this work, we will only be considering the 1D case so the y and z terms will disappear.

2) $$\frac{d^2P}{dt^2} = c^2 \frac{d^2P}{dx^2}$$

For this work, we will be looking at a medium transition at $x = 0$. For simplicity, we will set $c = 1$ for $x < 0$ and $c = 2$ for $x \geq 0$. The system's input will be a right traveling sine wave with an amplitude of 1 and a wavelength of 2. We will assume that there is no dissipation of the sound wave.

Solution Process

In order to solve this problem, we will be using d'Alembert's solution of the wave equation. This is a solution method can be used to solve the one-dimensional wave equation by making a change of variables and then solving by integration. The full derivation is beyond the scope of this work. We will begin with the solution taking the form:

3) $$P(x,t) = A(x-ct) + B(x+ct)$$

Where A and B represent arbitrary functions to be solved for based on the input and boundary conditions. This solution can be interpreted as representing the wave equation's solution as a superposition of right traveling and left traveling waves. This will be convenient for our scenario.

We will now represent the solution as a piecewise function with the same domain conditions as our wave speed, that is, we will have a solution of the form:

4) $$P(x,t) = \begin{cases} P_1(x,t), & x < 0 \\ P_2(x,t), & x \geq 0 \end{cases}$$

Where both $P_1(x,t)$ and $P_2(x,t)$ will have the form of equation 3. This allows us to rewrite P(x,t) in this form:

5) $$P(x,t) = \begin{cases} A(x - c_1 t) + B(x + c_1 t), & x < 0 \\ C(x - c_2 t) + D(x + c_2 t), & x \geq 0 \end{cases}$$

We can simplify this by understanding what will happen in this scenario. When our input wave reaches the transition, it can either be transmitted to the new medium or it can be reflected into the same medium but traveling in the opposite direction. This means that, for our input of a right traveling wave, our full solution can be represented with three waves: A will represent the input wave, B will represent the reflected wave, and C will represent the transmitted wave. Because the source is considered to be at x => -∞, there will be no left traveling wave to the right of the boundary and D will be 0.

Applying Conditions

There are three main conditions that must be maintained at the boundary:

i. The frequency of the wave cannot change

ii. $P_1(0,t) = P_2(0,t)$

iii. $\frac{dP_1}{dt}(0,t) = \frac{dP_2}{dt}(0,t)$

In order to apply the first condition, it is useful to first understand the general form of a traveling sine wave.

6) $$y = A \sin\frac{2\pi(x-ct)}{\lambda}$$

In the form shown in equation 6, A is simply the wave's amplitude. It is important to remember that the frequency of the wave is defined as the wave speed "c" divided by the wavelength. Putting each of our functions in the form of a traveling sine wave and applying condition i puts our solution in the form:

7) $$P(x,t) = \begin{cases} \sin\left(\frac{2\pi(x-t)}{2}\right) + R\,sin(\frac{2\pi(x+t)}{2}), & x < 0 \\ T\sin(\frac{2\pi(x-2t)}{4}), & x \geq 0 \end{cases}$$

We see in equation 7 that the wavelengths on either side of the boundary must be different to maintain a constant frequency accounting for the change in wave speed. Also, R and T are constants that we have not yet determined. They can be solved for by applying conditions ii and iii. This case yields values of R = -1/3 and T = 4/3 so our final solution is:

8) $$P(x,t) = \begin{cases} \sin\left(\frac{2\pi(x-t)}{2}\right) - \frac{1}{3}sin(\frac{2\pi(x+t)}{2}), & x < 0 \\ \frac{4}{3}\sin(\frac{2\pi(x-2t)}{4}), & x \geq 0 \end{cases}$$

CONCLUSIONS

The most interesting part of this result is that the wave in the medium that the sound wave originates from is more chaotic than the medium it transitions to because of the interference with the reflected wave. This can be seen in Figure 1 as the amplitude in the left medium varies with time while the amplitude in the right medium stays constant.

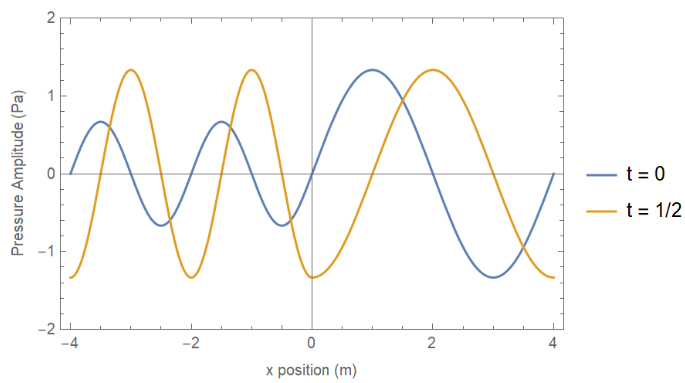

Figure 1: Right moving Pressure wave changing mediums shown at two different times

While the specific scenario used in this work used arbitrary constants chosen for convenience, the process used to get to the solution can be easily generalized to a variety of constants and inputs. This would allow for better prediction of how sound waves will behave as they transition between mediums and can inform design decisions in acoustics and other applications where pressure waves are important.

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APPENDIX

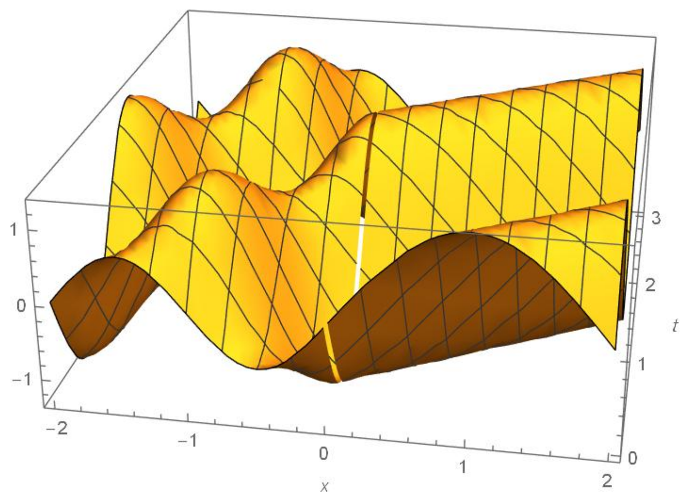

Figure 2: 3 dimensional representation of a pressure wave changing in time as it changes mediums

