

BÉZIER CURVE FITTING METHOD FOR EXISTING TURBINE BLADE DESIGN

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ABSTRACT

Design of axial flow turbomachinery is a critical step in the iterative process of gas turbine development. Existing designs for the aerodynamic blade profiles are not flexible enough to use with advanced design analysis and automation tools. A Bézier curve fitting approach is proposed as a tool to improving the existing design for future use and development. An optimization technique is used to determine the control points for the approximated curves while maintaining tangency and curvature at the connections. Results are provided and the drawbacks to the curve fitting and optimization approach are discussed.

NOMENCLATURE

$P(t)$ – Bézier curve of order n

$B_n^i(t)$ – i th Bernstein basis polynomial of degree n

INTRODUCTION

Geometric design of axial flow turbomachinery is critical to the development of a gas turbine engine. Often the aerodynamic blade design is the inner loop of the overall iterative engine design process. As a result, methods for creating blade profiles must be robust and flexible in order to efficiently provide many different configurations during the design cycle. Past methods for developing blade profiles often involve complex parameterization schemes which use arcs, circles, and polynomials for geometry creation. Many of these methods require large data sets to describe the profile. In addition, they do not provide simple, intuitive parameters for the designer to control and frequently result in additional smoothing and geometric changes during the final stages of design. This paper proposes a method for creating an improved blade definition from existing legacy blade designs. Bézier curves are used to fit an existing data set while maintaining tangency and curvature between the curve connections.

BACKGROUND

Bézier curves were developed in the 1960's by Pierre Bézier while working as an engineer for the Renault automobile company. Bézier curves have since become a popular method for creating parametric curves. Bézier curves have wide application including PostScript font definition. However, since Bézier's initial development of these simple parametric curves, additional refinements to the have been made in the definition of B-splines and NURBS. These curves are both generalizations of the basic Bézier curve and have specific application in advanced computer aided design, analysis, and manufacturing.

BÉZIER CURVE DEFINITION

A Bézier curve of n order is defined by the following equation:

$$P(t) = \sum_{i=0}^n B_i^n(t) \cdot P_i$$

where P_i represents the set of $n + 1$ control points, t is a single-valued parameter which varies from 0 to 1, and $B_i^n(t)$ is the blending function

$$B_i^n(t) = \binom{n}{i} \cdot (1-t)^{n-i} \cdot t^i, \quad i = 0, 1, \dots, n$$

In the blending function formula $\binom{n}{i}$ is the binomial

coefficient defined as $\frac{n!}{i!(n-i)!}$. The blending functions are

referred to as the i th Bernstein basis polynomial of degree n . Figure 1 shows four different examples of Bézier curves of degree one through four.

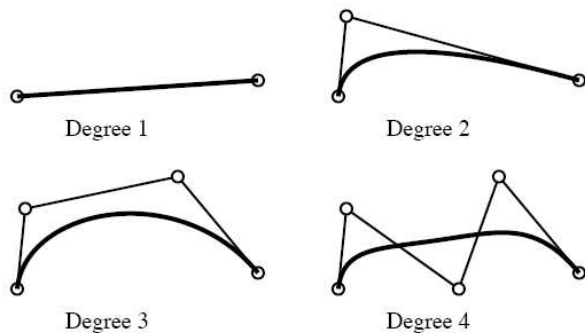


Figure 1. Examples of Bézier curves of different degree

From the above equations and figure it can be seen that the blending functions act as the link between the control points and the points on the Bézier curve. In each case the first and last control points lie on the actual curve while the other control points form a control envelope for the curve.

Often a single Bézier curve is insufficient for describing a complex profile. In this case it is common to use several Bézier curve segments joined together. Different parametric continuity conditions exist for joining multiple Bézier curves. In order for two curves to actually join they must meet at a junction point. This is termed as *order zero* continuity. For the Bézier curve definition the last control point of the first curve and the first control point of the second curve must be equal. A second type of continuity is tangency or continuity of slope. This *first order* continuity is imposed by setting the last two control points of the first curve and the first two control points of the second curve in a straight line and the length of the two line segments equal to each other. *Second order* continuity, or curvature continuity, further restricts freedom of the control points. In this case continuity is guaranteed by equating the ratio $\frac{h}{a^2}$ for each curve where h is the perpendicular distance from the third control point and the first leg (the line segment between the first and second control points) of the control polygon and a is the length of the first leg of the control polygon.

Bézier curve continuity can also be of the geometric type. Geometric continuity is less restrictive and allows for increase flexibility in the variation of the magnitude of the length of the control polygon legs. While curves meeting the parametric continuity conditions may also be said to be geometrically continuous, the reverse is not true.

BÉZIER CURVE FITTING METHOD

The Bézier curve approach was used to fit second order continuous curves to existing turbine blade design data. The existing design provided a data set of points for each of the four curve segments shown in Figure 2.

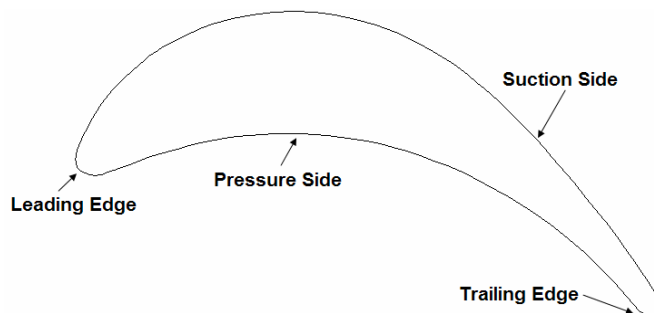


Figure 2. Turbine blade curve segment locations

Each curve segment was defined by an arbitrary number of data points. The break points for each curve were determined by the spacing of the data points. Both the leading edge and trailing edge were constructed tightly spaced points that created a clear division between the curve segments.

In order to fit the Bézier curves to the existing data an optimization based model was constructed. The t -values for the blending functions were determined by calculating the distance between the data points and setting the t -values to approximate the same intervals. For the optimization routine the control points were selected as the design variables. Constraints were set to control the continuity at the connections. The first and second order continuity was controlled using the following differential equations:

$$y_1'(1) = C_1 \cdot y_2'(0)$$

$$y_1''(1) = C_1^2 \cdot y_2''(0) + C_2 \cdot y_2'(0)$$

where y_1 and y_2 are the Bézier curve function evaluated at some t -value and C_1 and C_2 are two constants used as design variables in the optimization. By setting these functions as constraints the tangency and curvature at the junctions was fixed. The optimization routine was then used to minimize the error between the existing data set and the best fit Bézier curve.

RESULTS

The optimization technique provided accurate curve fitting for the pressure and suction side curves. The resulting Bézier curves and the existing data set are shown in figures 3 and 4 for the pressure and suction sides respectively.

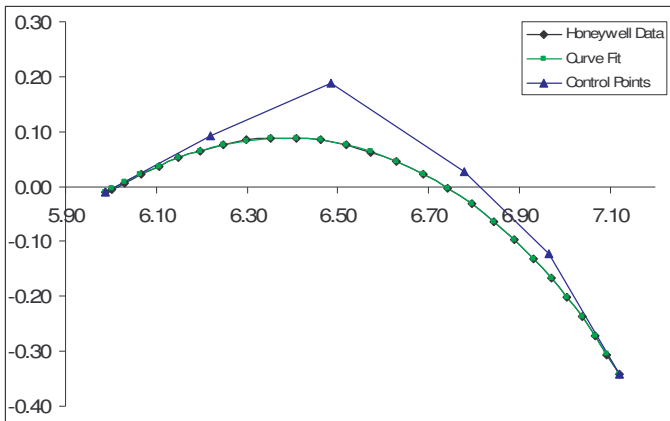


Figure 3. Pressure side curve with Bézier curve approximation

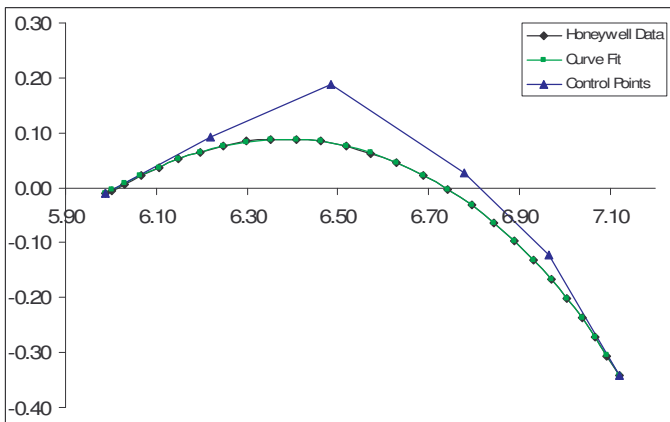


Figure 4. Suction side curve with Bézier curve approximation

From the above charts it can be seen that the Bézier curve approximation closely matches the existing data set while still maintaining tangency and curvature at the connection points.

Although the Bézier curve fitting strategy worked well for the larger pressure and suction side curves, the leading and trailing edge segments were not able to fit the data as closely without violating the continuity constraints. Figures 5 and 6 show the optimized Bézier curves as well as the legacy blade data. For both curves the continuity constraints forced the Bézier approximation away from the data set in order to provide a feasible solution. This was the result of a large change in curvature between the relatively flat and pressure change and suction side curves and the tightly curved leading and trailing edges. Therefore, the optimization routine created noticeable error in order to satisfy continuity. Although the error in fitting these segments provided visibly different curves, the new parametric Bézier definition can be manipulated to provide a slightly modified geometry that would still function in future turbine blade design.

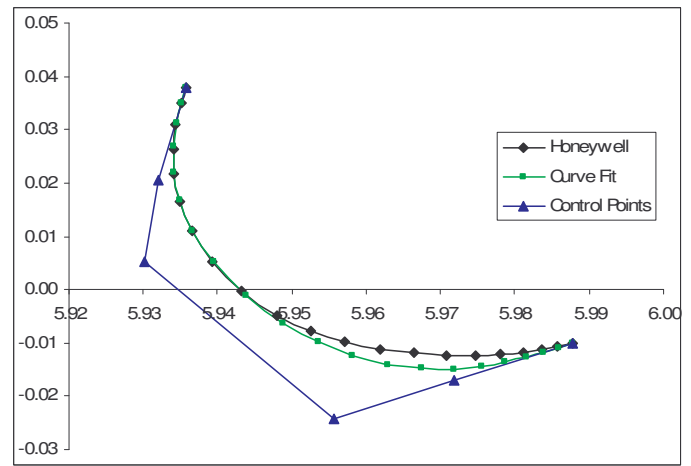


Figure 5. Leading edge curve with Bézier curve approximation

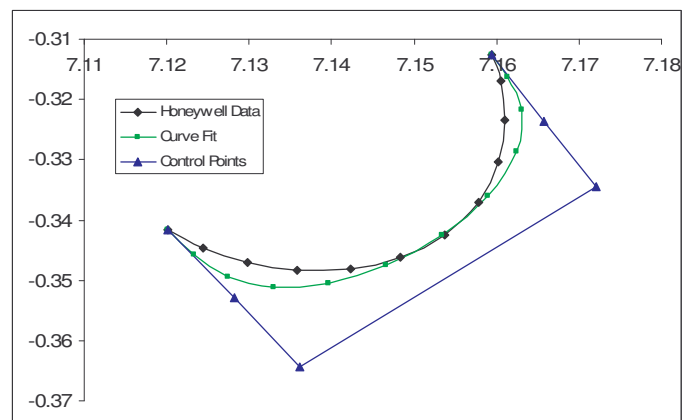


Figure 6. Trailing edge curve with Bézier curve approximation

CONCLUSIONS & RECOMMENDATIONS

A Bézier curve approximation for fitting existing turbine blade design data is a useful method for creating a robust, parametric geometry definition. This approach provided a simple method for optimizing the curve fit based on the continuity at the junctions.

Although the optimization technique was able to provide accurate results for the curve fit, it did require the designer to frequently manipulate the optimization parameters and run multiple routines. Slight changes in the location of the control points would drive the optimization to undesirable local optimums.

Additional methods for determining the control points should be considered. A least squares fit would provide a closed form solution for calculating the control points at a give set of t -values. The error could then be refined using a simple Newton-Raphson algorithm. In addition, the legacy data used for the curve fitting was not guaranteed to maintain tangency and curvature at all locations. Therefore, resulting solutions should not be ranked on the error alone.

REFERENCES

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APPENDIX

Excel curve fitting tool available upon request.